Likely chaotic transitions of large-scale fluid flows using a stochastic transport model

Valentin Resseguier, Etienne Mémin, Bertrand Chapron

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Motivations for deriving random fluid dynamics models

- Rigorously identified sudgrid dynamics effects
- Injecting likely small-scale dynamics
- Quantification of modeling errors:

  especially for Data assimilation: ensemble forecasts

- **Predicting possible distinct scenarios**

Contents

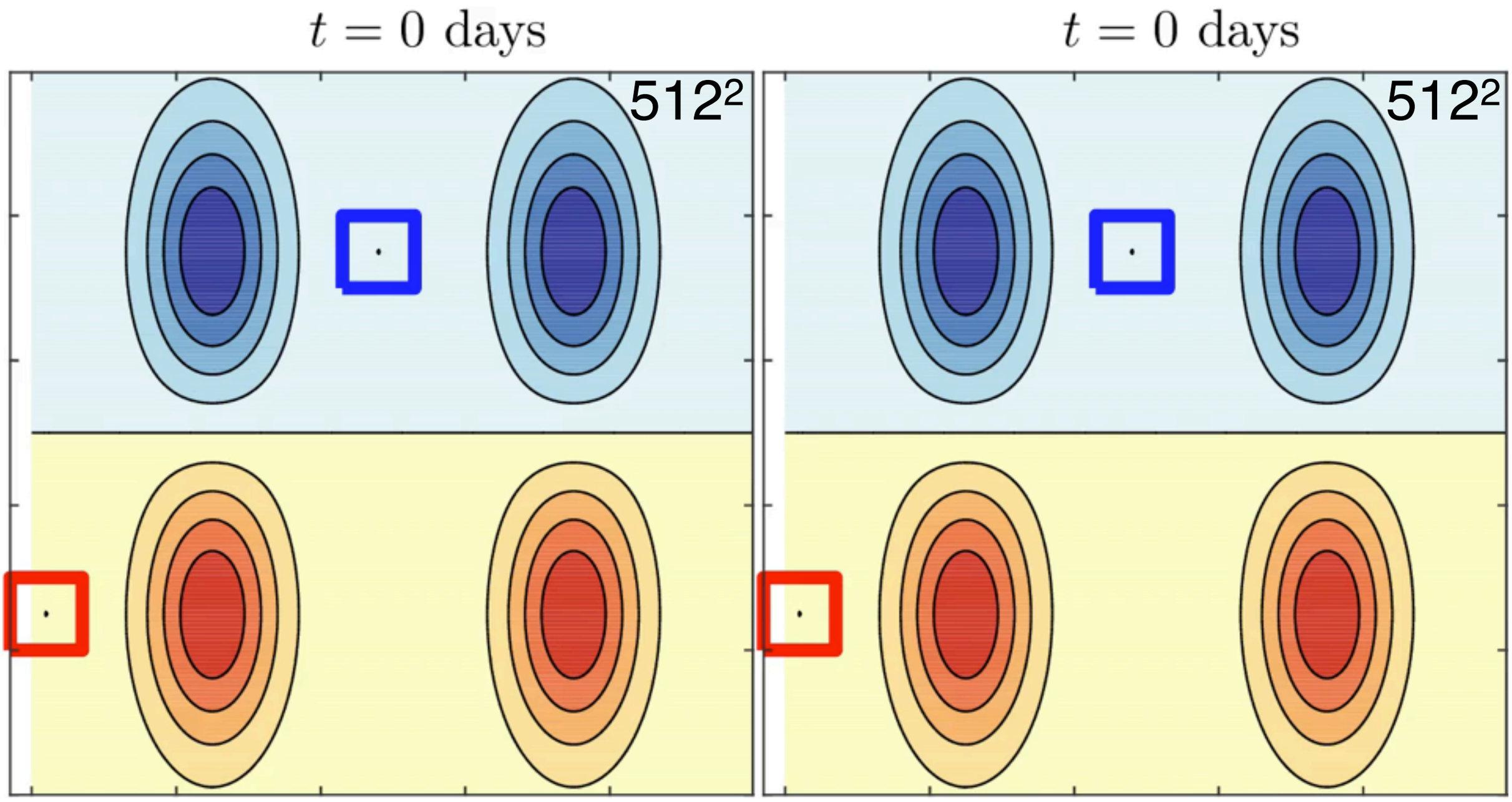
- Chaotic transitions
- Dynamics under location uncertainty
- Ensemble of simulations

Chaotic transitions

in SQG dynamics

Reference flow: deterministic SQG 512^2

Initial condition 1 → Scenario 1



Reference flow: deterministic SQG 512^2

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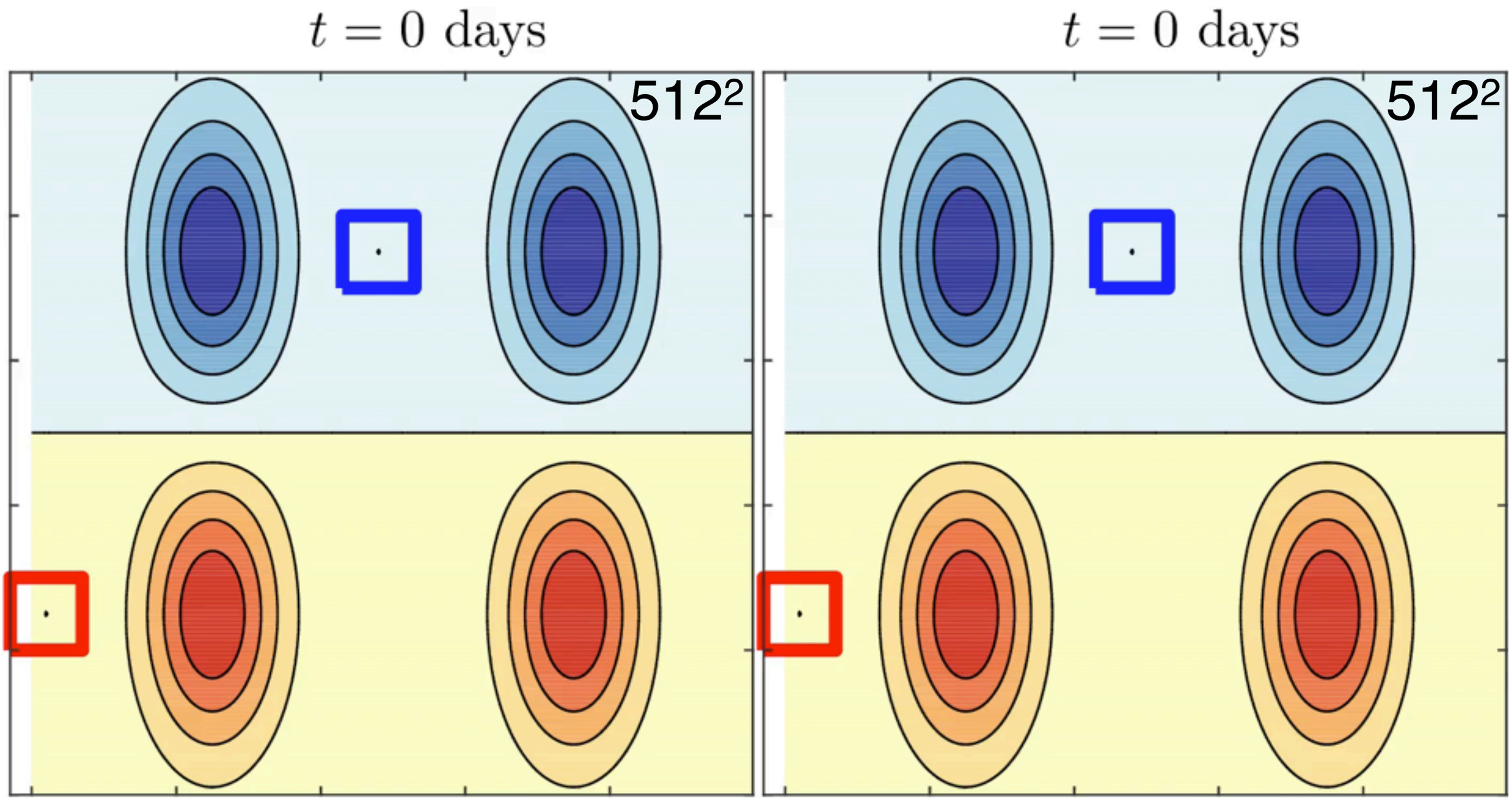
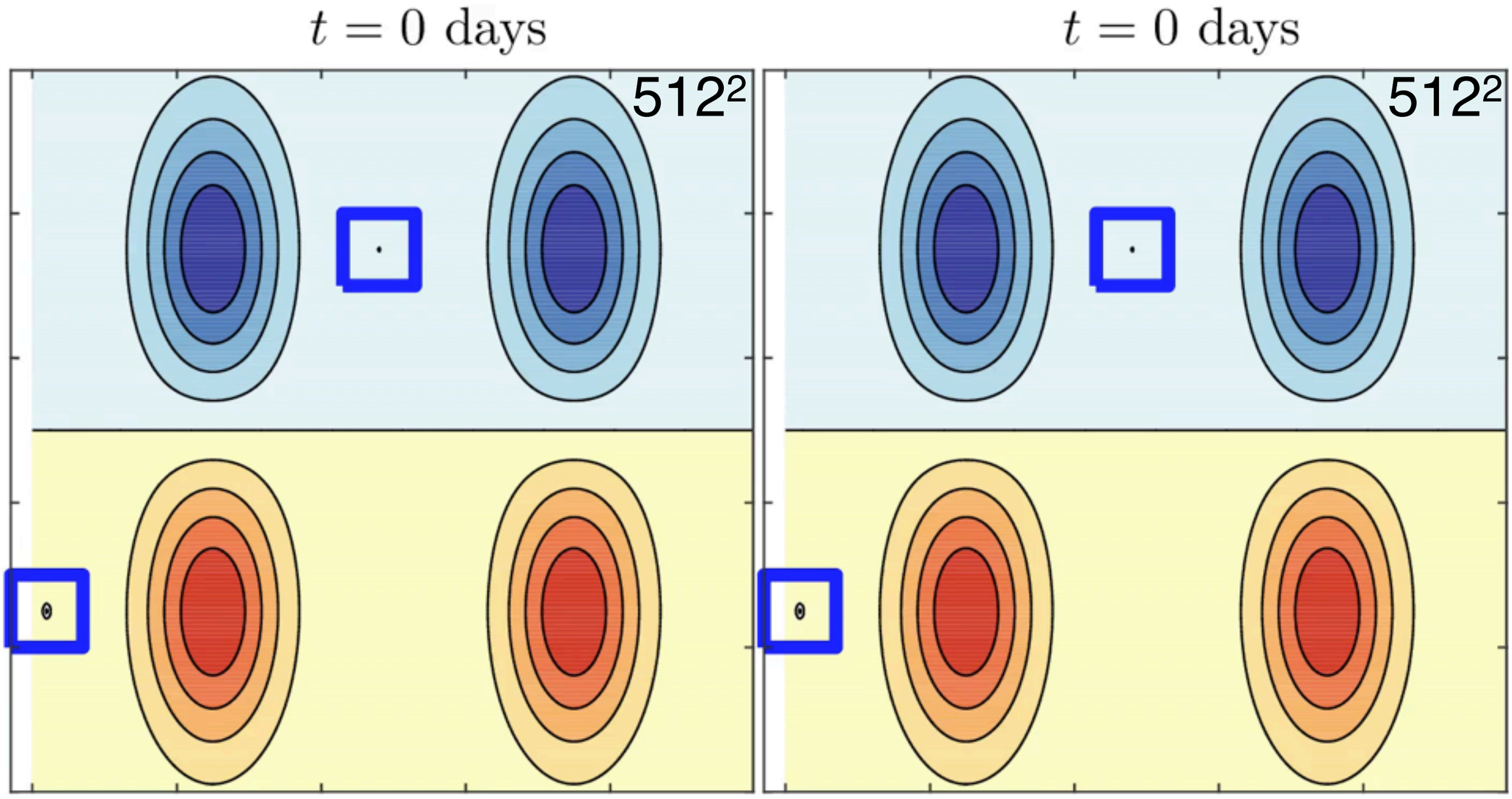


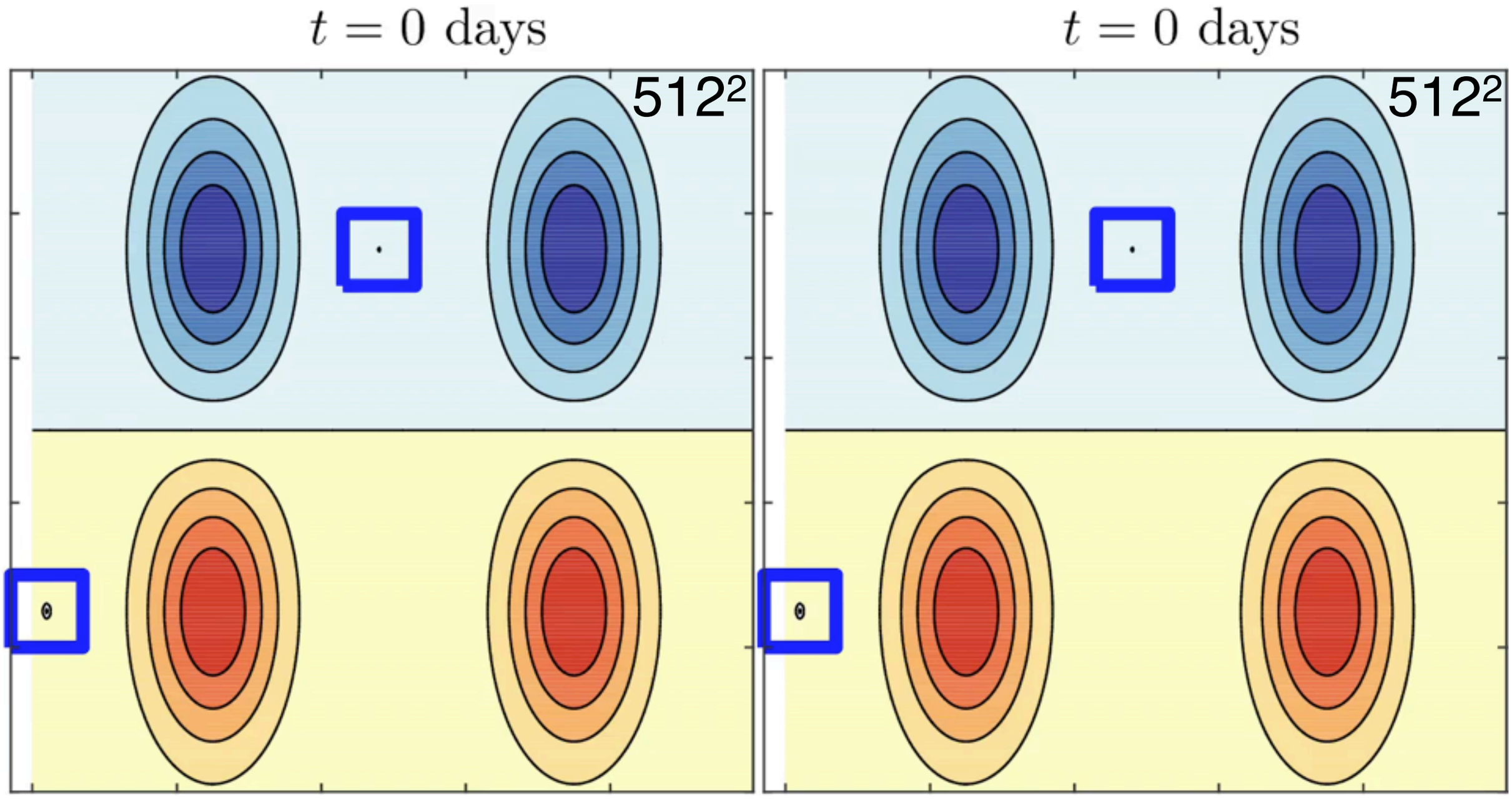
Reference flow: deterministic SQG 512^2

Initial condition 2 → Scenario 2



Reference flow: deterministic SQG 512^2

Initial condition 2 → Scenario 2

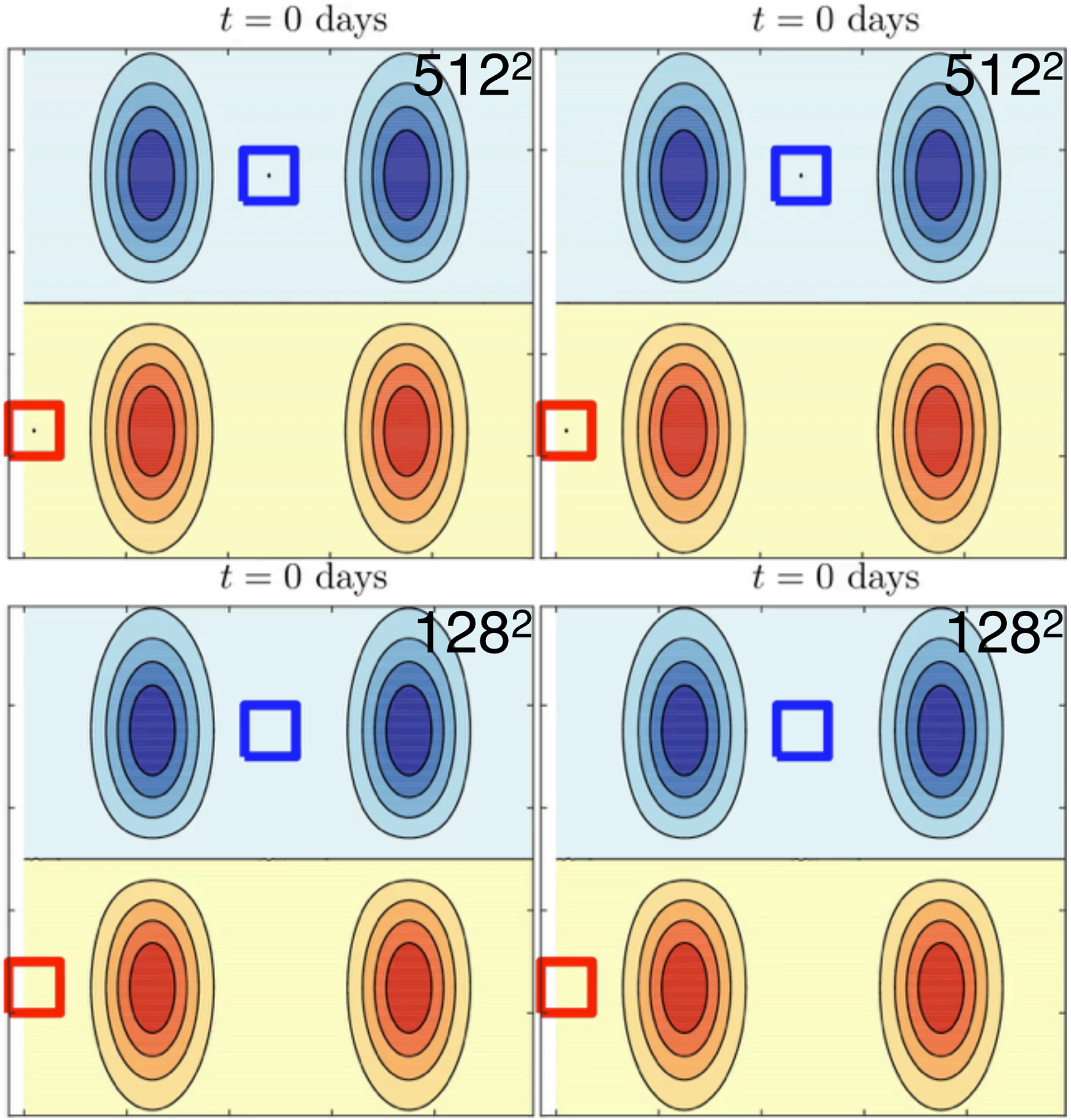


Reference flow: deterministic SQG

512^2 versus 128^2

Initial condition 1

?



Reference flow: deterministic SQG

512^2 versus 128^2

Initial condition 1

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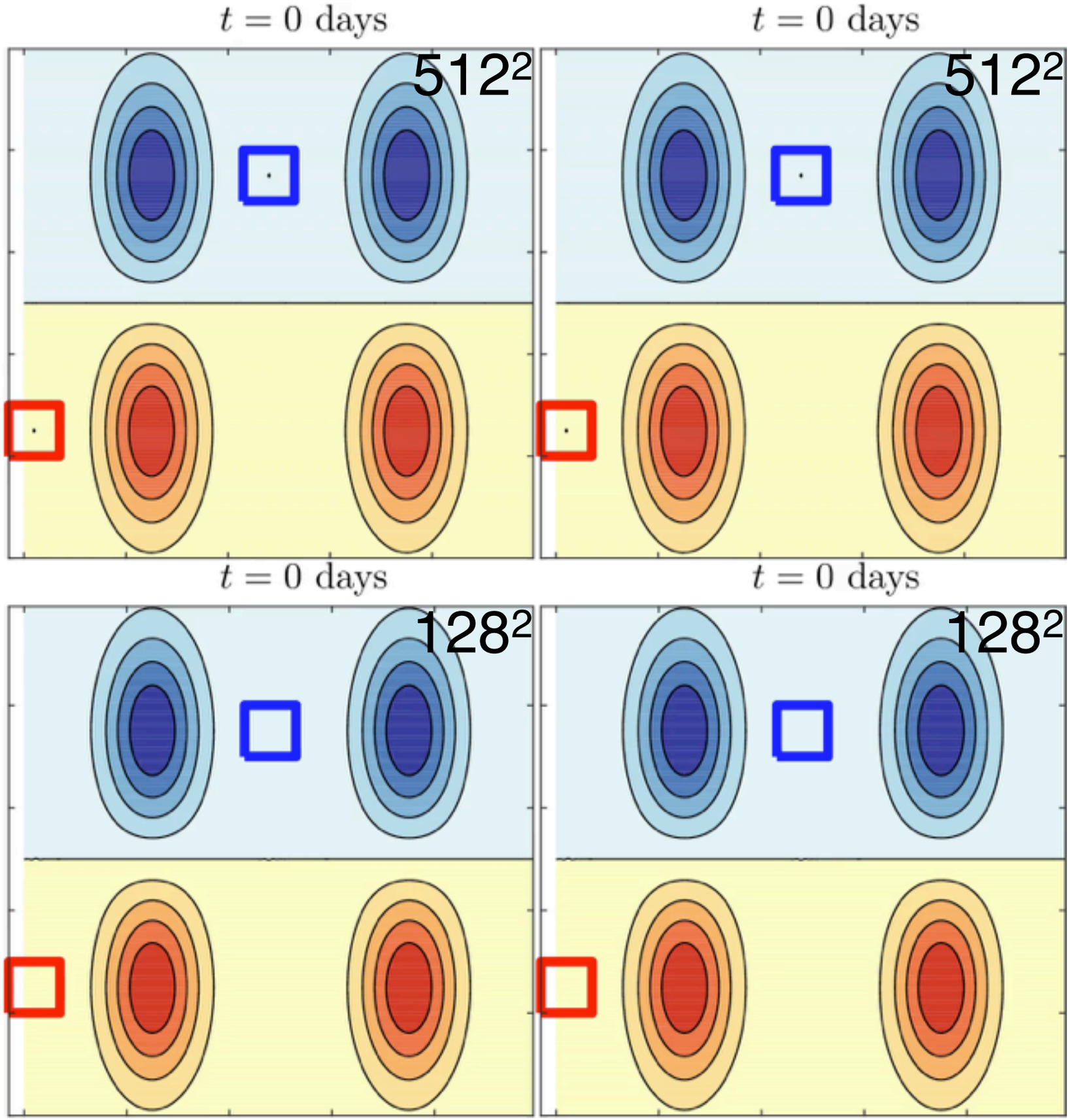


Dynamics under location uncertainty

Random equations

- Random initial conditions → Underdispersive + need large ensemble
- Arbitrary Gaussian forcing → Adding energy + wrong phase
- Averaging, homogenization → Assumptions and energy issues
- Adding white random velocity → $\boldsymbol{v} = \boldsymbol{w} + \boldsymbol{\sigma}\dot{\boldsymbol{B}}$

Advection of tracer Θ

$$\frac{D\Theta}{Dt} = 0$$

Advection of tracer Θ

Advection of tracer Θ

$$\partial_t \Theta + \boldsymbol{w}^\star \cdot \boldsymbol{\nabla} \Theta + \boldsymbol{\sigma} \dot{\boldsymbol{B}} \cdot \boldsymbol{\nabla} \Theta = \boldsymbol{\nabla} \cdot \left(\frac{1}{2} \boldsymbol{a} \boldsymbol{\nabla} \Theta \right)$$

Advection of tracer Θ

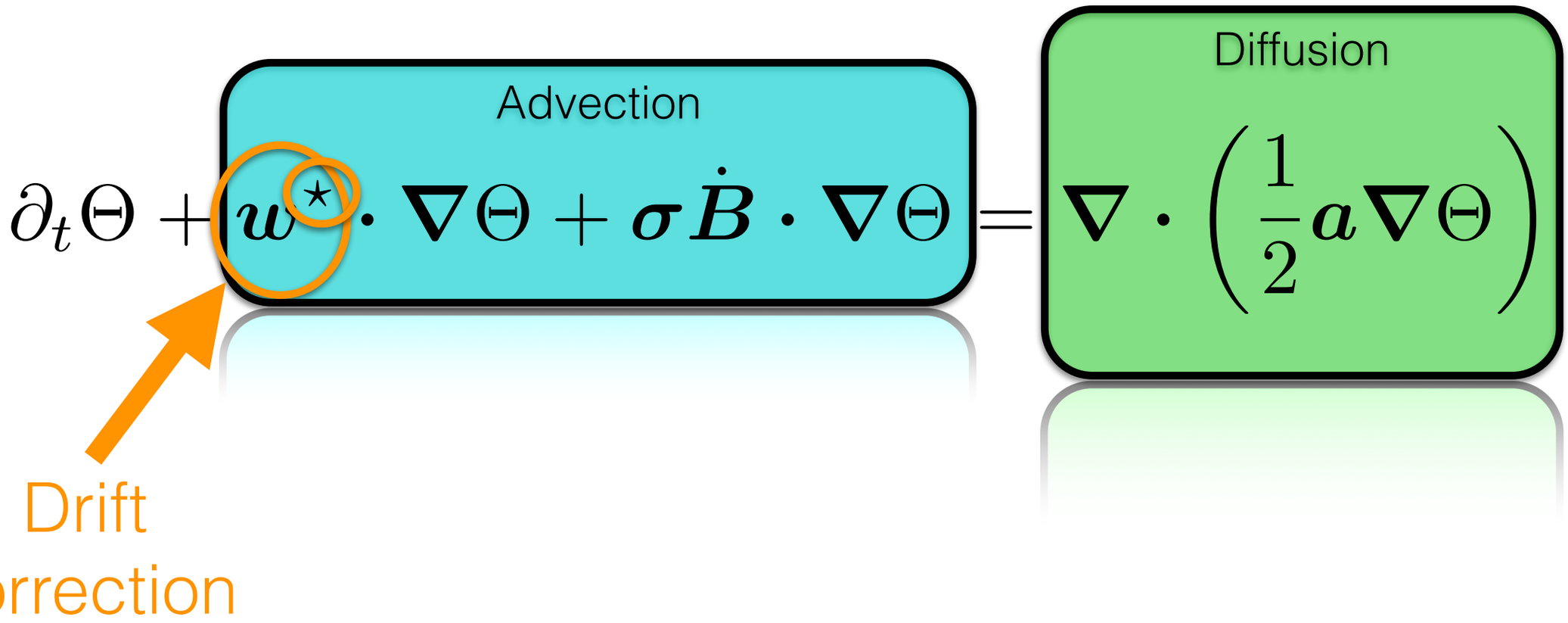
$$\partial_t \Theta + \boxed{\boldsymbol{w}^\star \cdot \boldsymbol{\nabla}\Theta + \boldsymbol{\sigma}\dot{\boldsymbol{B}} \cdot \boldsymbol{\nabla}\Theta} = \boldsymbol{\nabla} \cdot \left(\frac{1}{2} \boldsymbol{a} \boldsymbol{\nabla}\Theta \right)$$

Advection

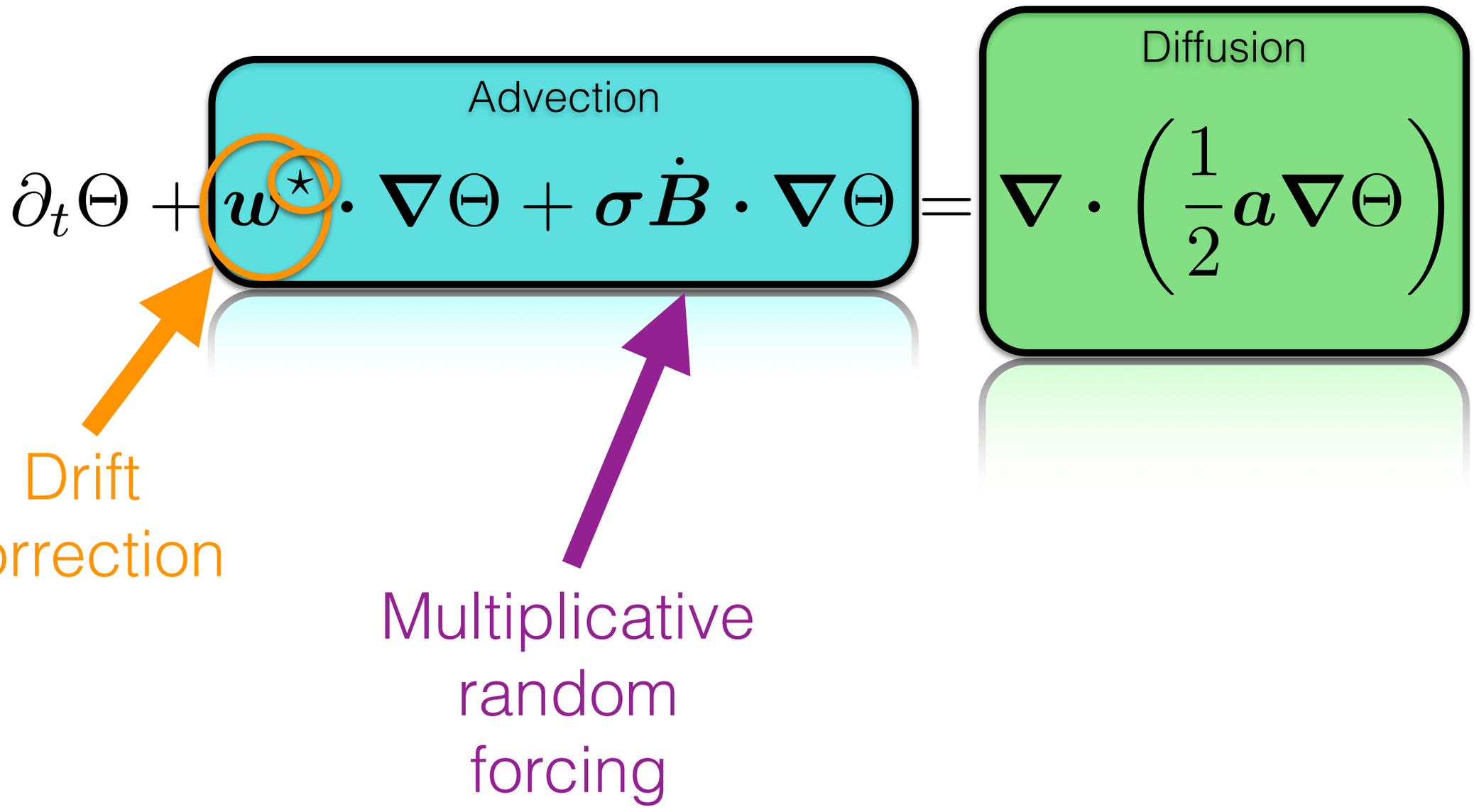
Advection of tracer Θ

$$\partial_t \Theta + \underbrace{\boldsymbol{w}^\star \cdot \boldsymbol{\nabla}\Theta + \boldsymbol{\sigma}\dot{\boldsymbol{B}} \cdot \boldsymbol{\nabla}\Theta}_{\text{Advection}} = \underbrace{\boldsymbol{\nabla} \cdot \left(\frac{1}{2} \boldsymbol{a} \boldsymbol{\nabla}\Theta \right)}_{\text{Diffusion}}$$

Advection of tracer Θ



Advection of tracer Θ



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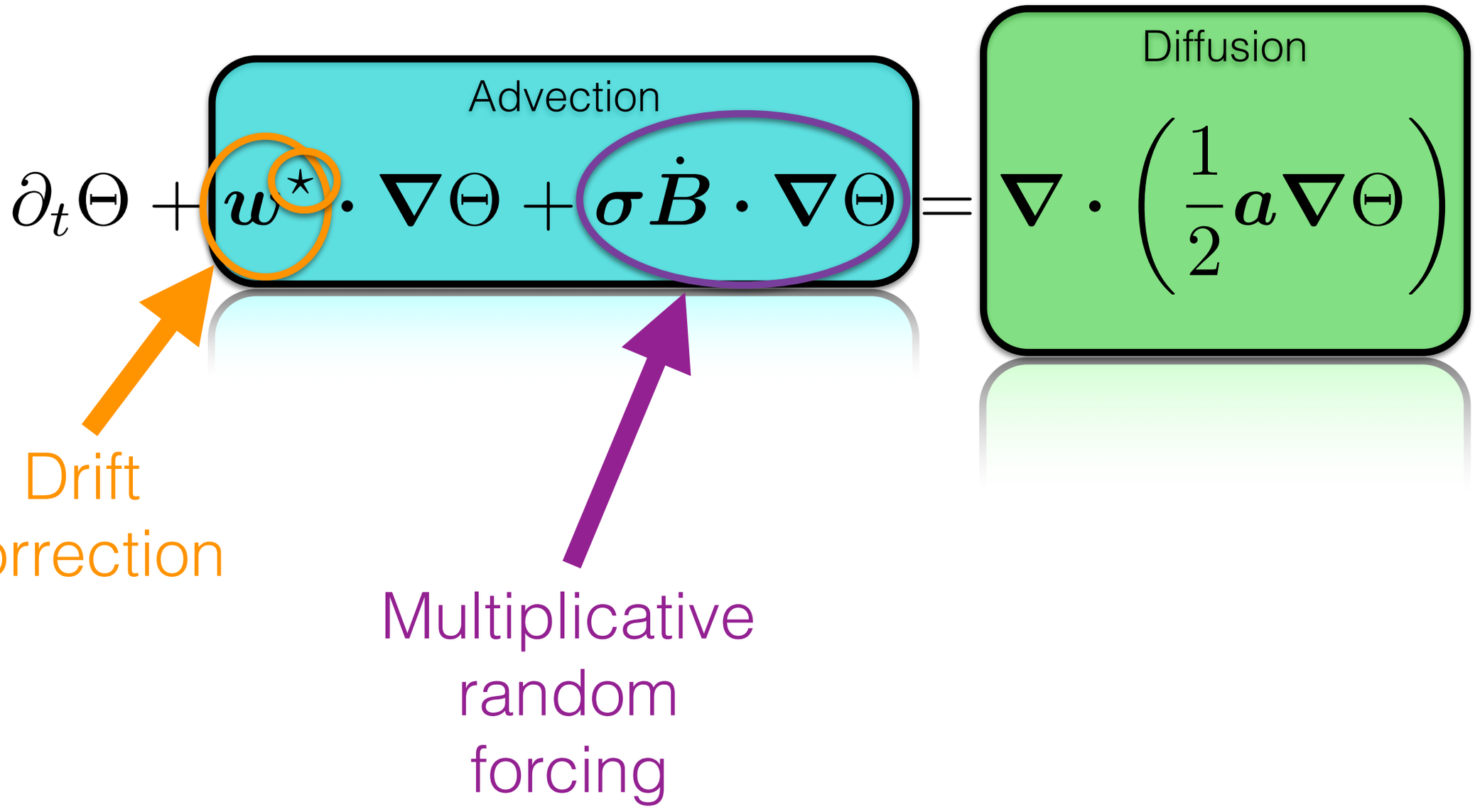
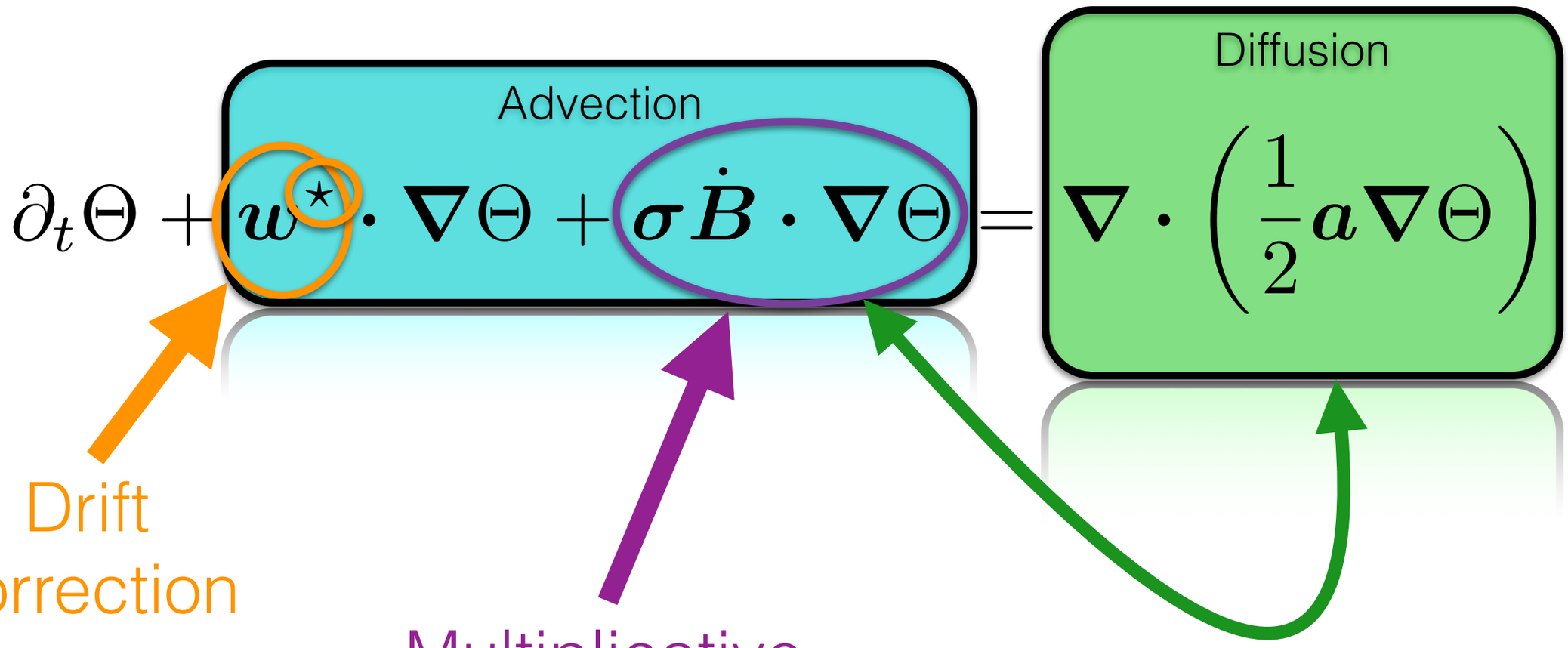
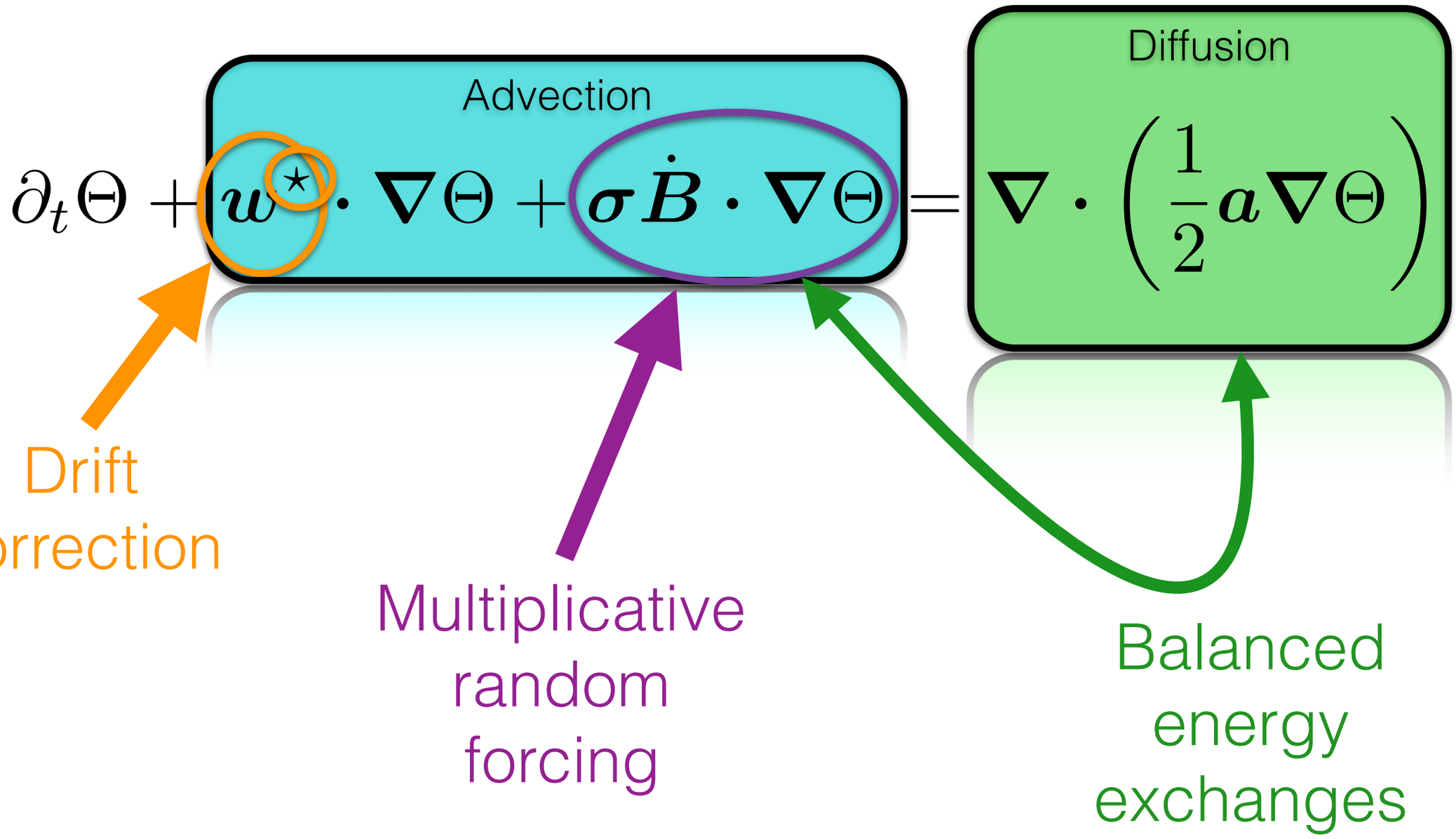


Advection of tracer Θ



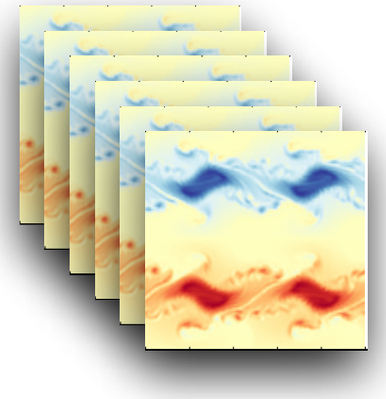
Advection of tracer Θ



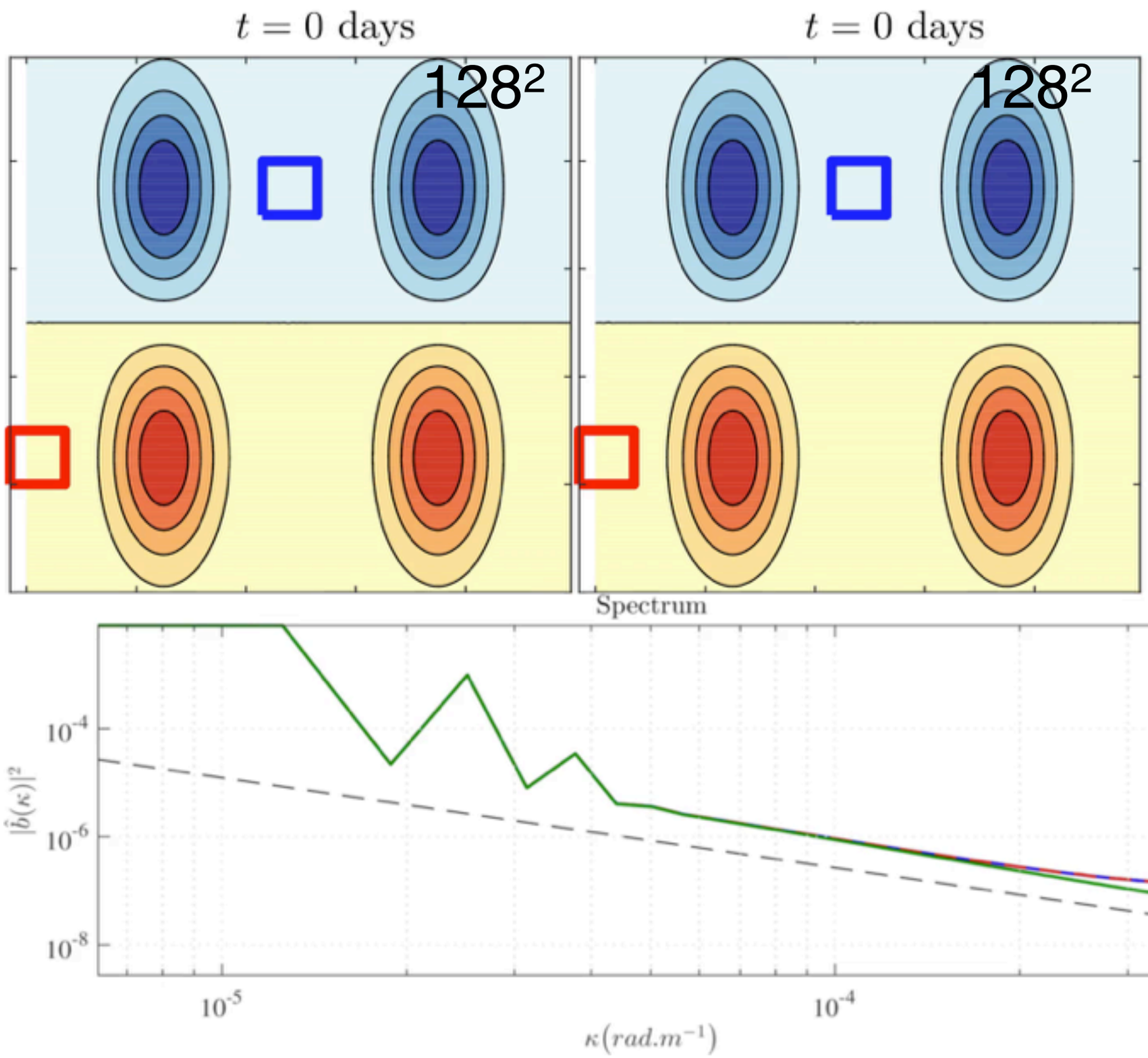
Ensemble of simulations

with SQG MU

Code available online

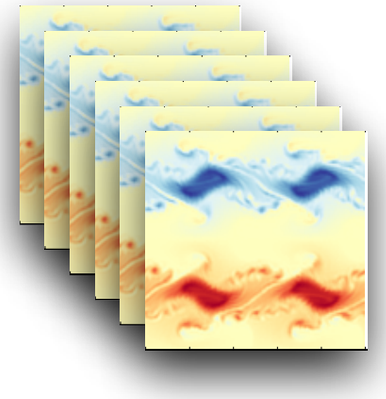


examples of realizations:



Spectrum

mean^2/variance:



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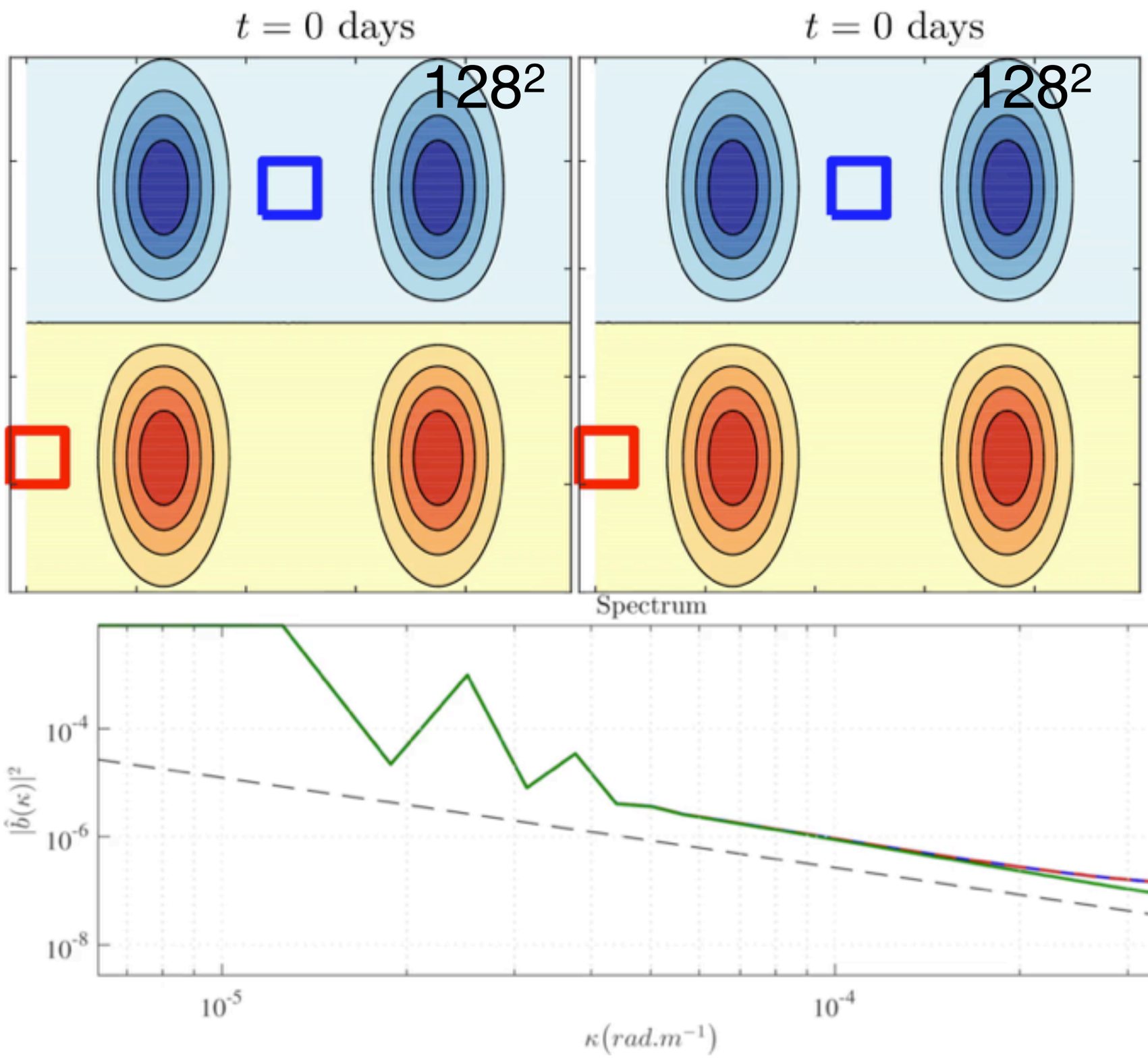

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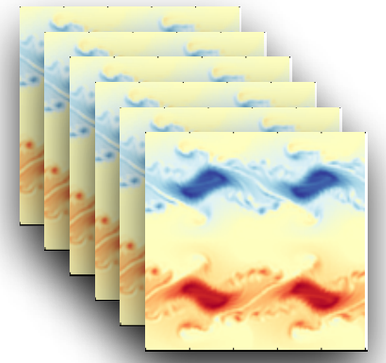
mean^2/variance:

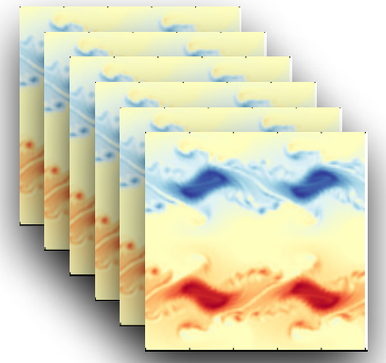


Visualization of the ensemble

At a fixed time t,

Principal Component Analysis (EOF)

$$\Theta^{(i)}(\boldsymbol{x},t) \approx \hat{\mathbb{E}}(\Theta)(\boldsymbol{x},t) + \sum_{n=1}^{N_{EOF}} c_n^{(i)}(t)\Psi_n(\boldsymbol{x},t)$$



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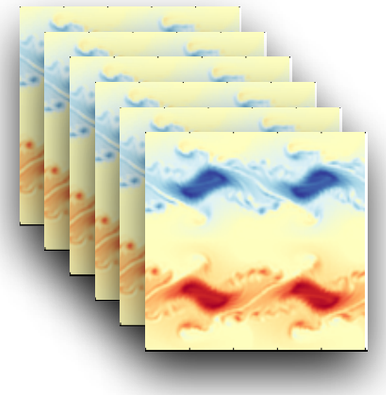
$\Theta^{(i)}$: i^{th} realization



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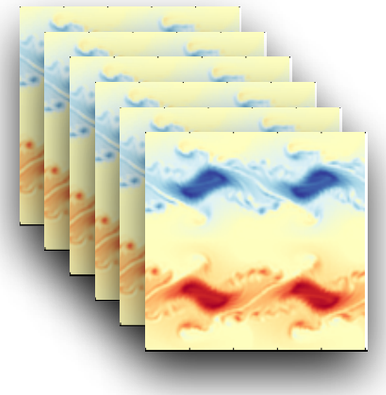
$\hat{\mathbb{E}}(\Theta)$: Mean



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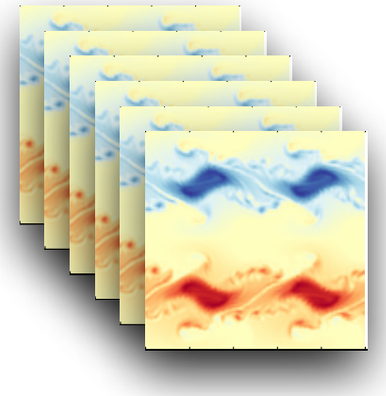
$\Theta^{(i)}$: i^{th} realization

$\hat{\mathbb{E}}(\Theta)$: Mean

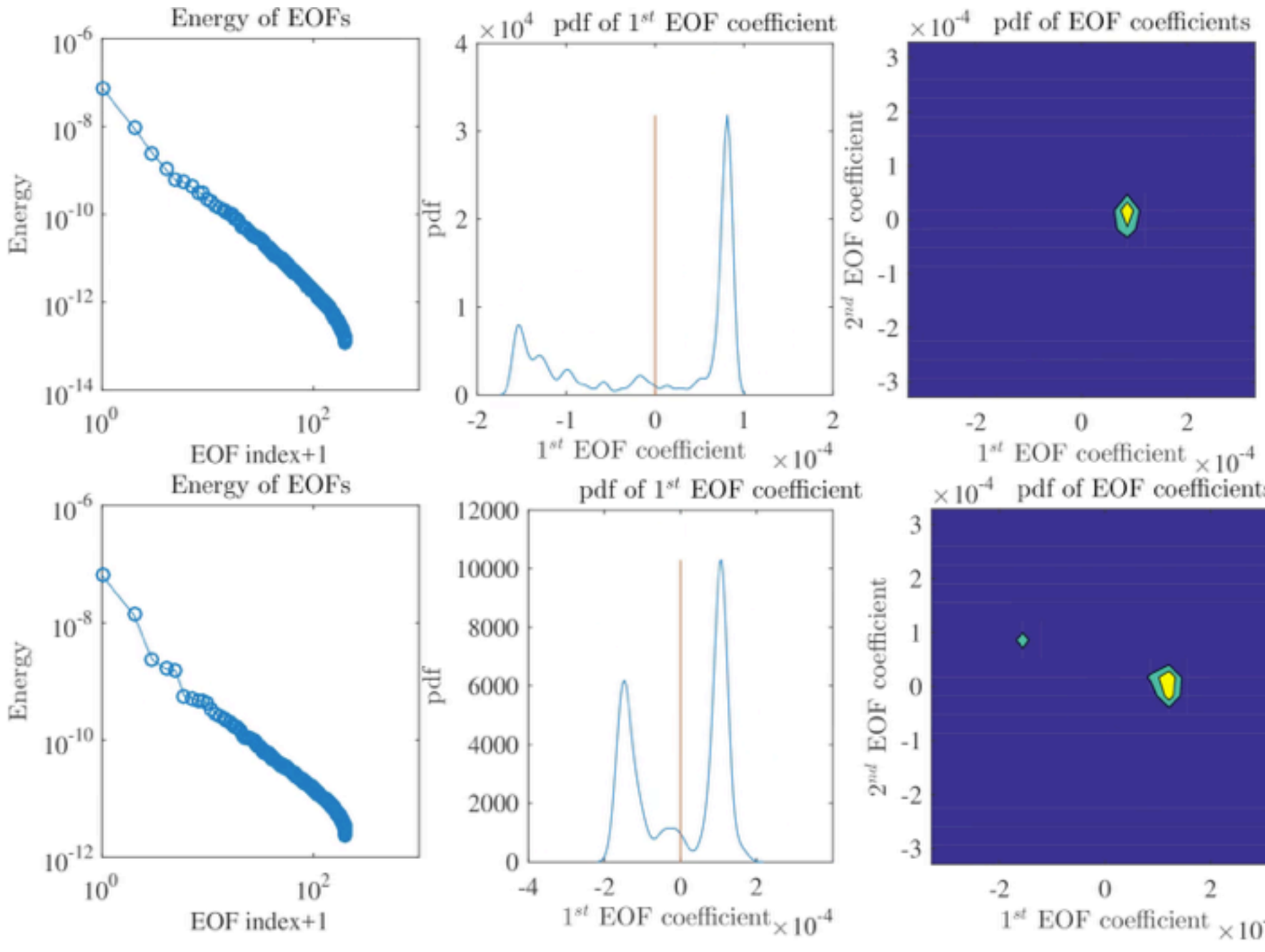
$c_n^{(i)}$: (random) EOF coefficient

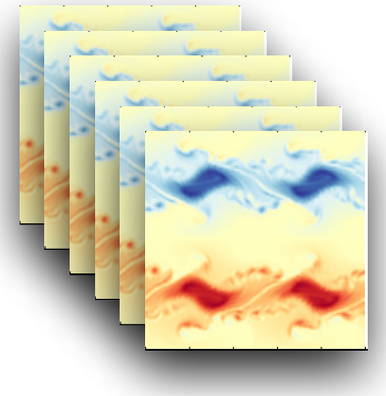


Visualization of the ensemble (200 realizations)

Random initial conditions

Under location uncertainty





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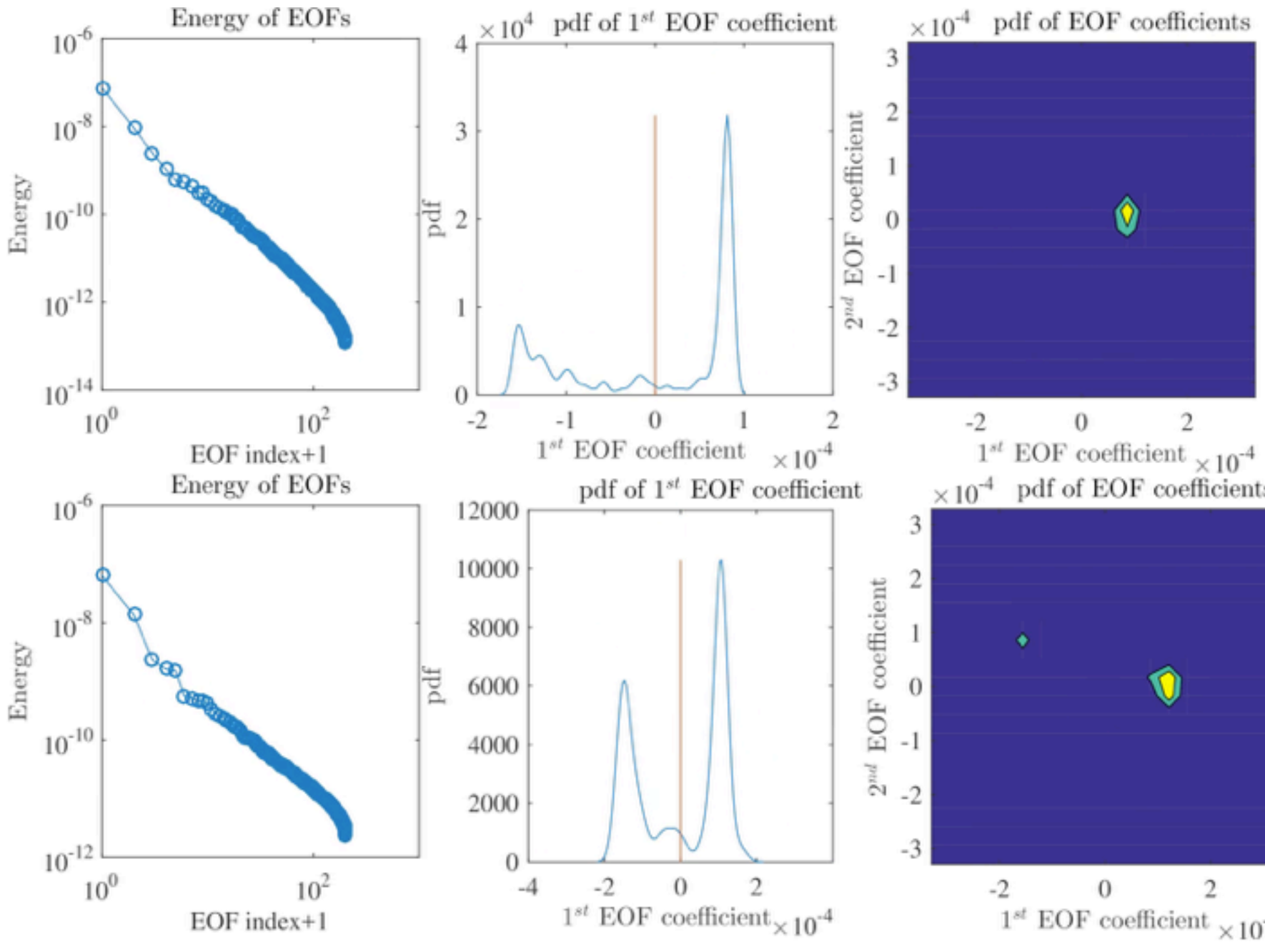
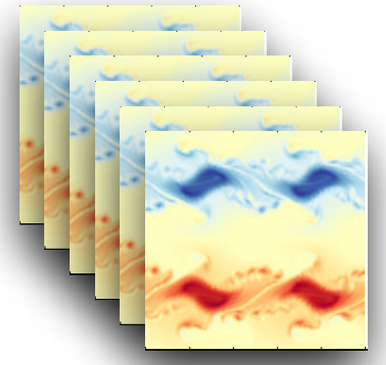
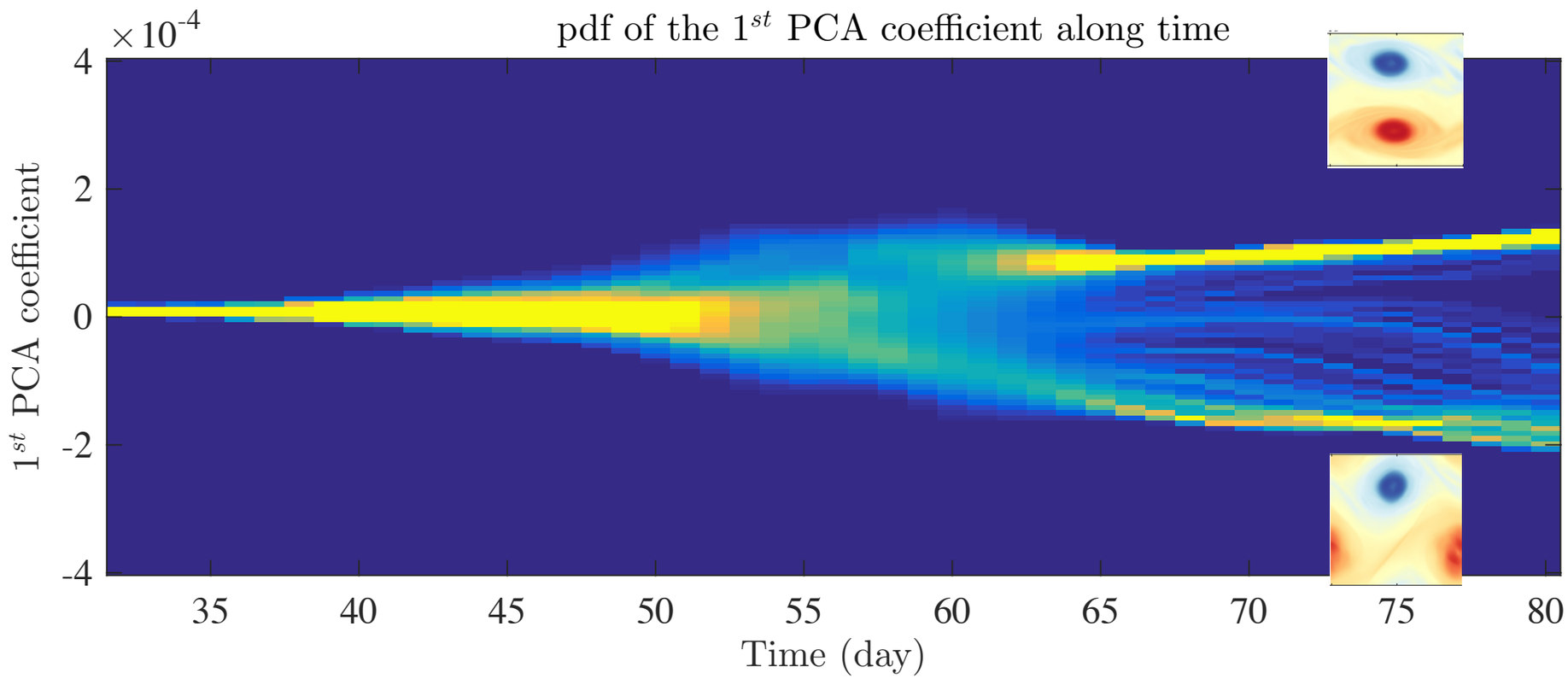
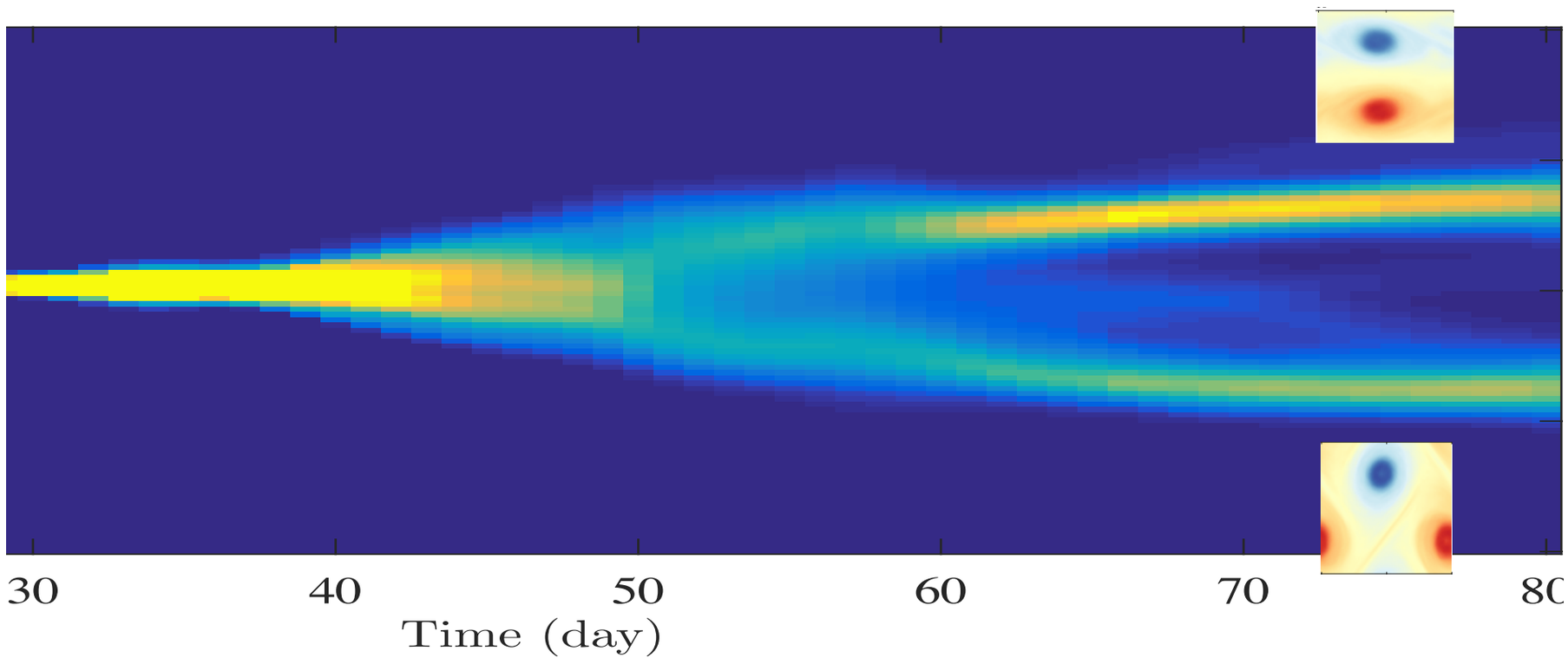




Random initial conditions



Under location uncertainty



Conclusion

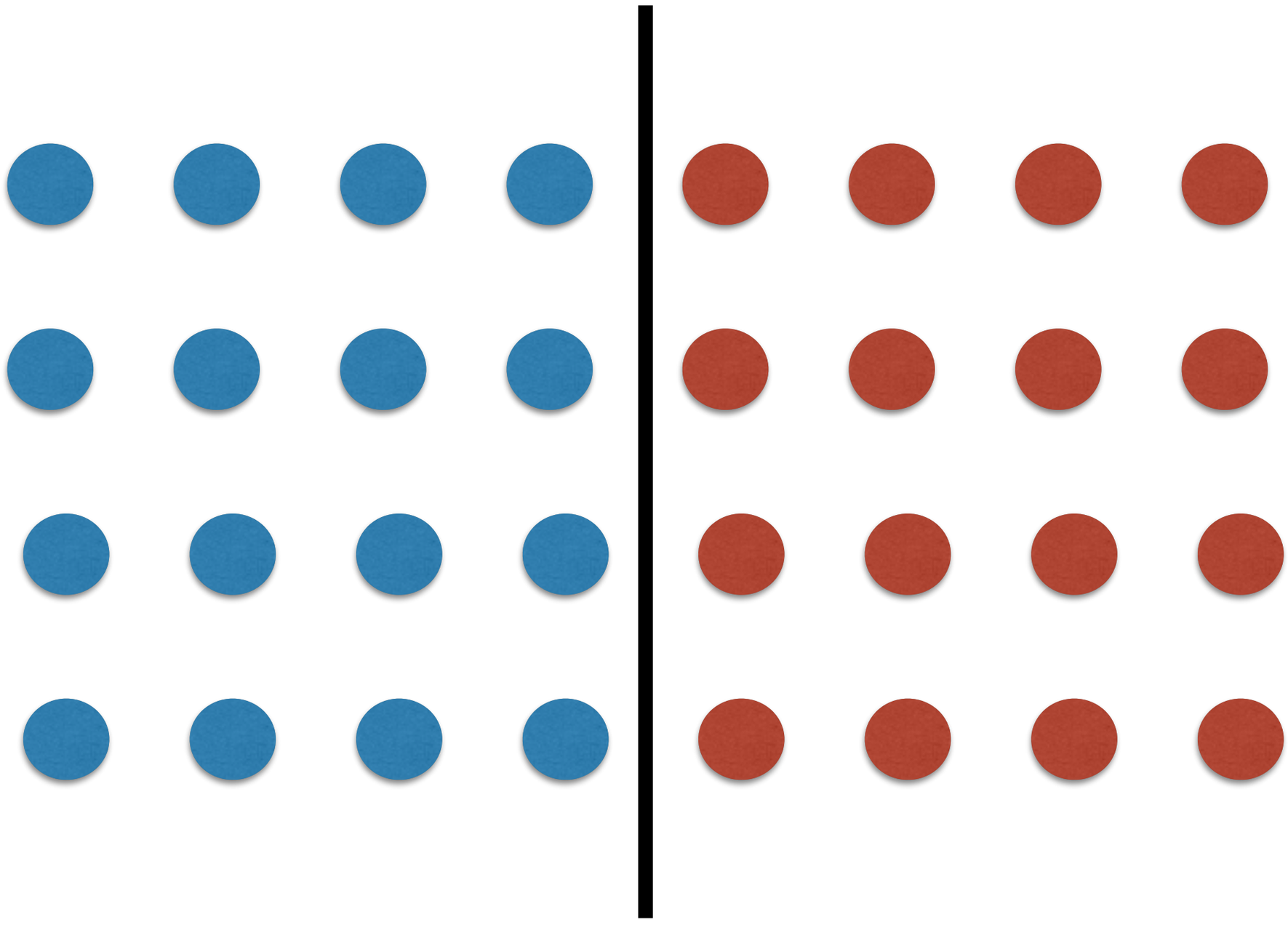
Conclusion

- Random transport applicable to any dynamics
- Link (inhomogeneous) diffusion, drift correction and multiplicative noise
- Predict likely scenarios with few realizations
- Other results:
    - Better small scales
    - Estimate positions and amplitudes of errors
    - Extreme events
    - Additional physical information

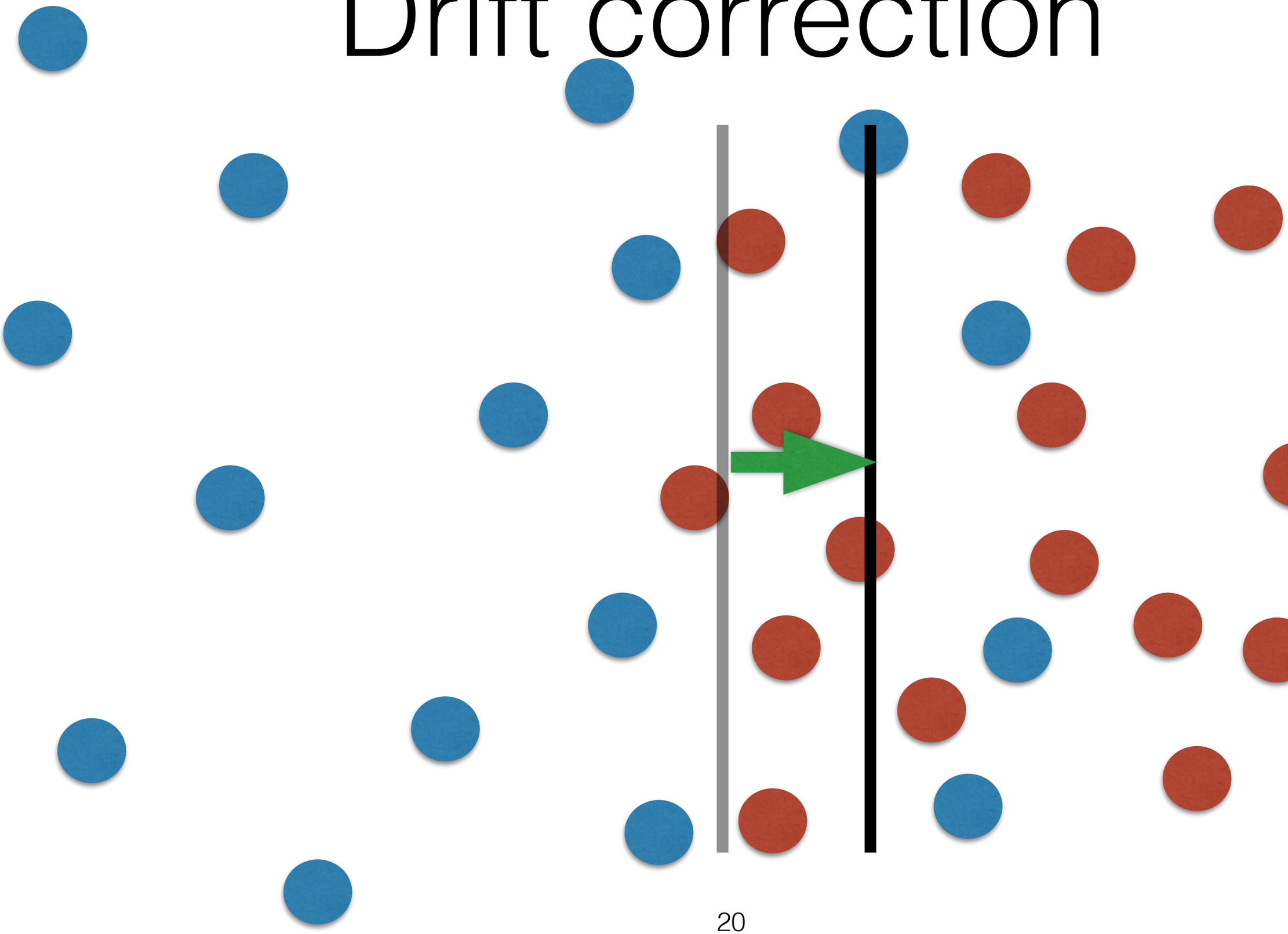
Thank you for your attention

Code SQG MU:
link from Fluminance website - V. Resseguier

Drift correction

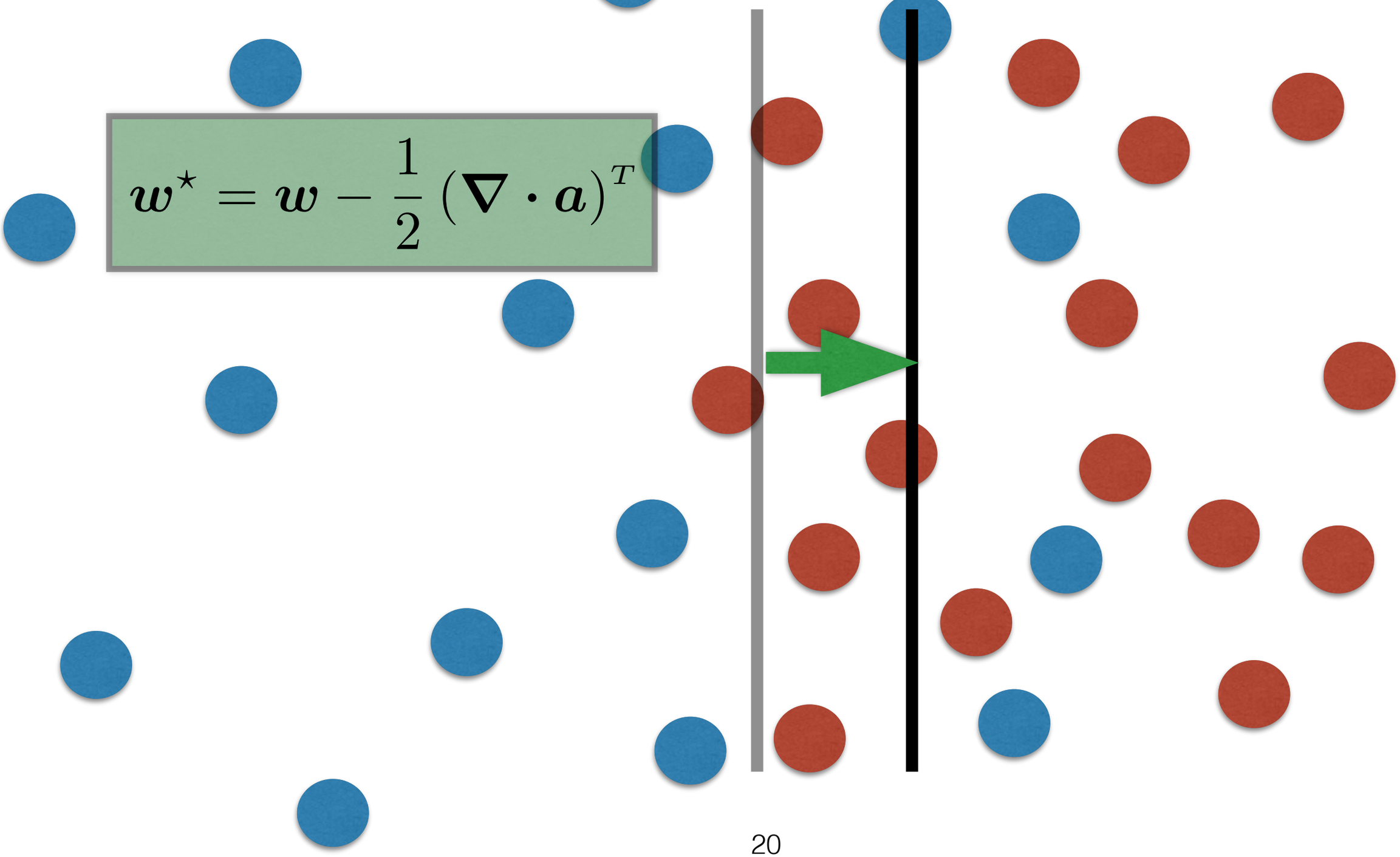


Drift correction



Drift correction

$$\boldsymbol{w}^{\star} = \boldsymbol{w} - \frac{1}{2}\left(\boldsymbol{\nabla} \cdot \boldsymbol{a}\right)^{T}$$



SQG under Moderate Uncertainty

SQG MU

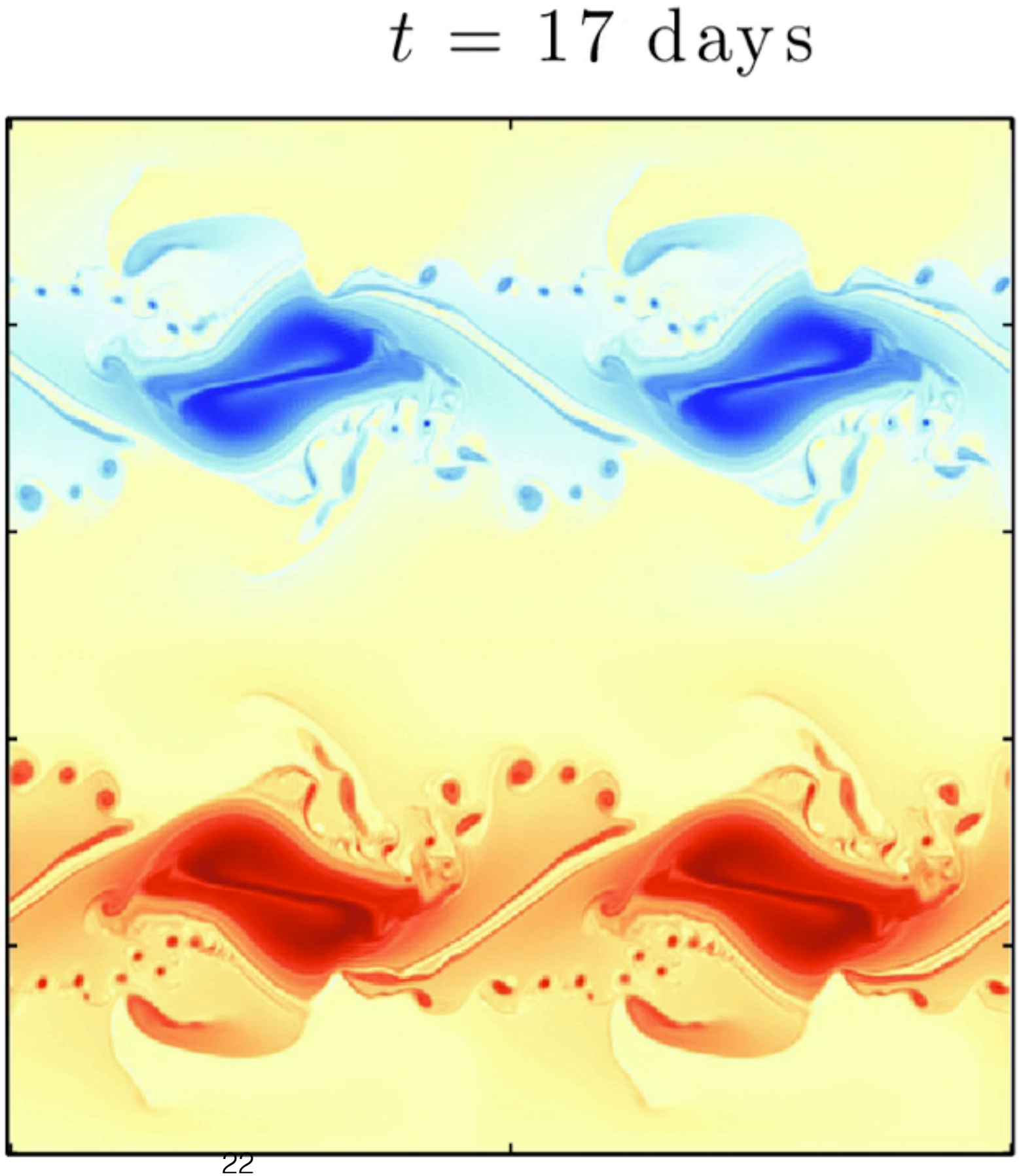
Code available online

Reference flow:

deterministic

SQG

512 x 512



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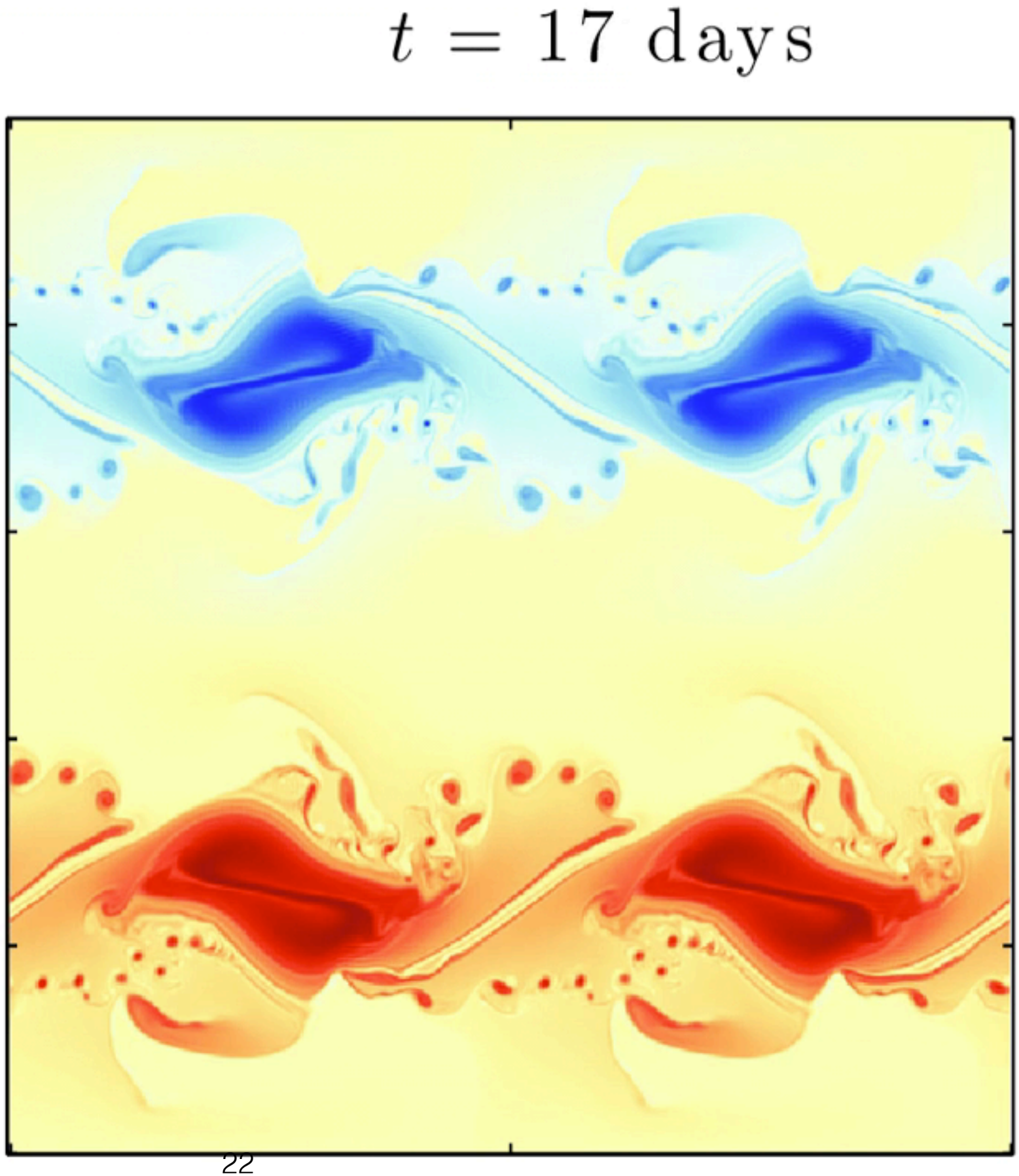
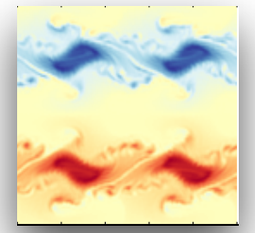
deterministic

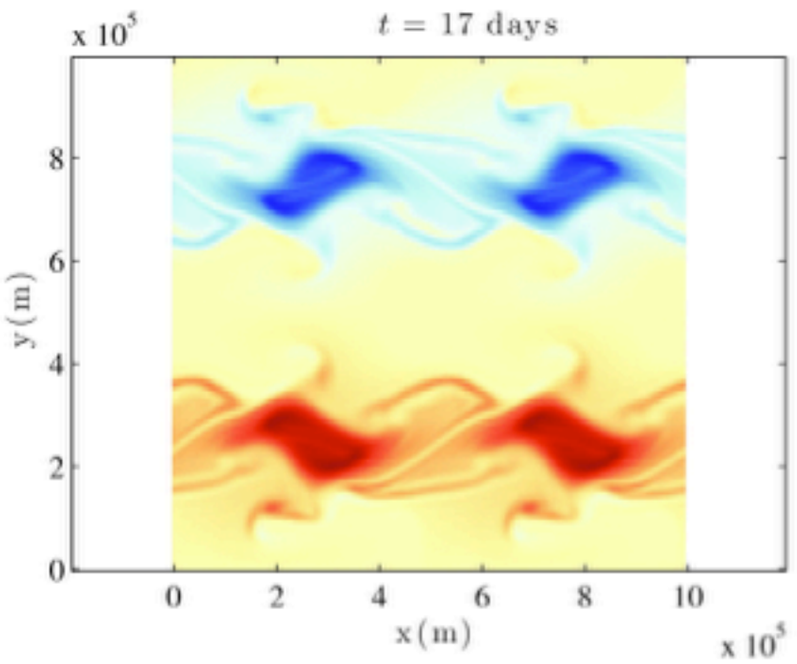
SQG

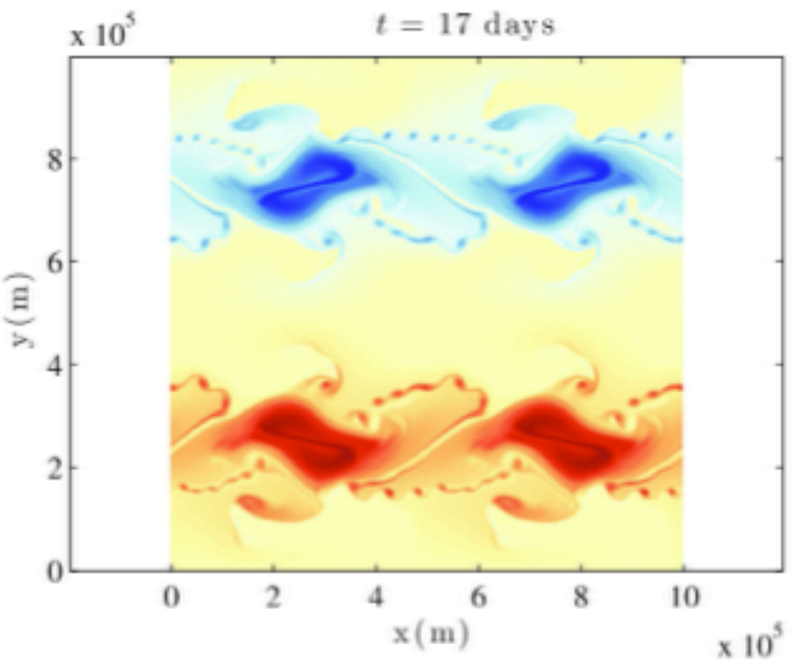
512 x 512

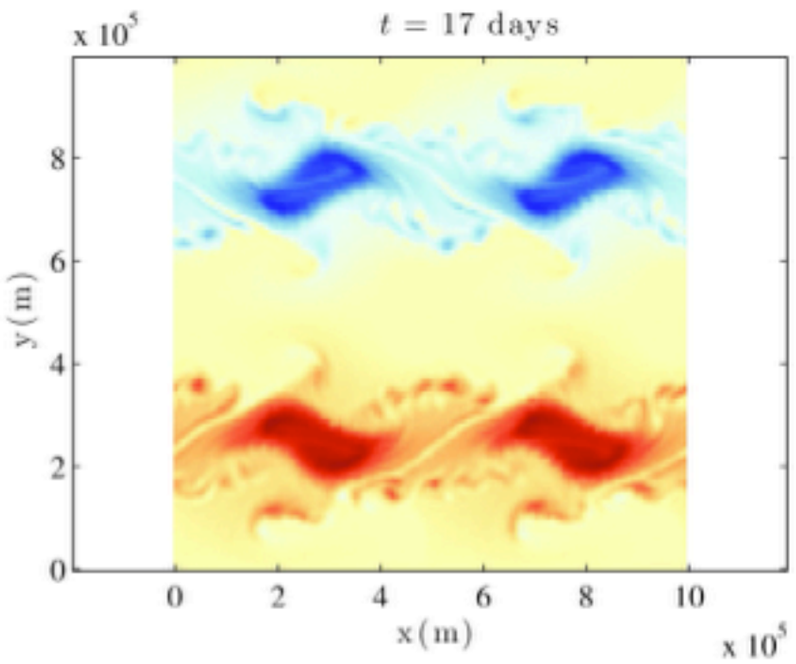


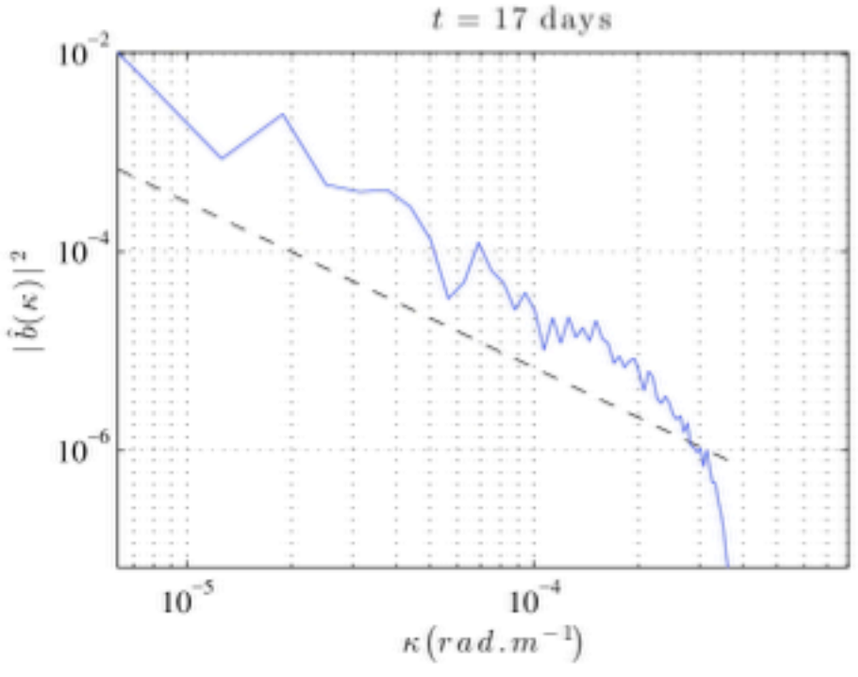


One realization

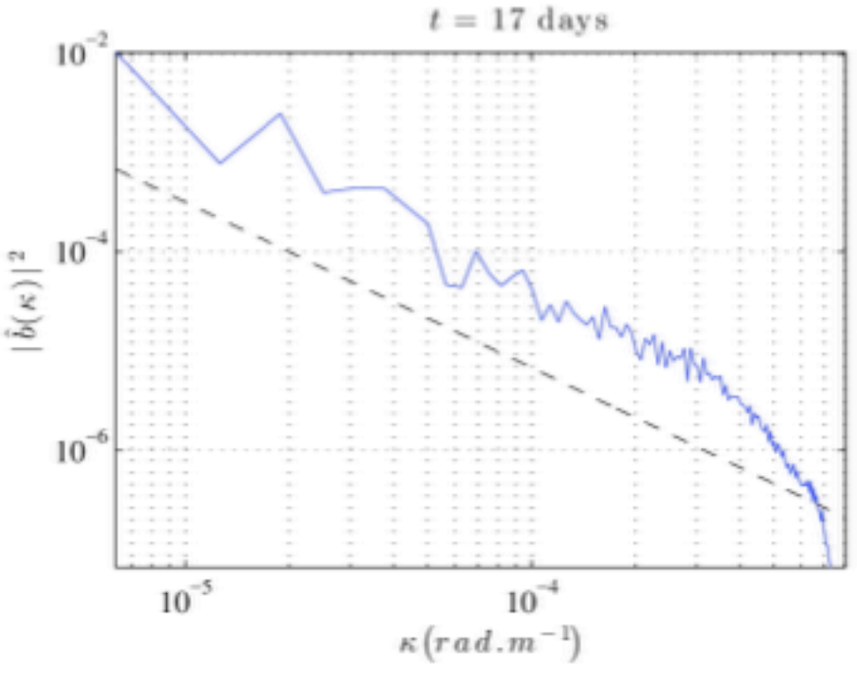




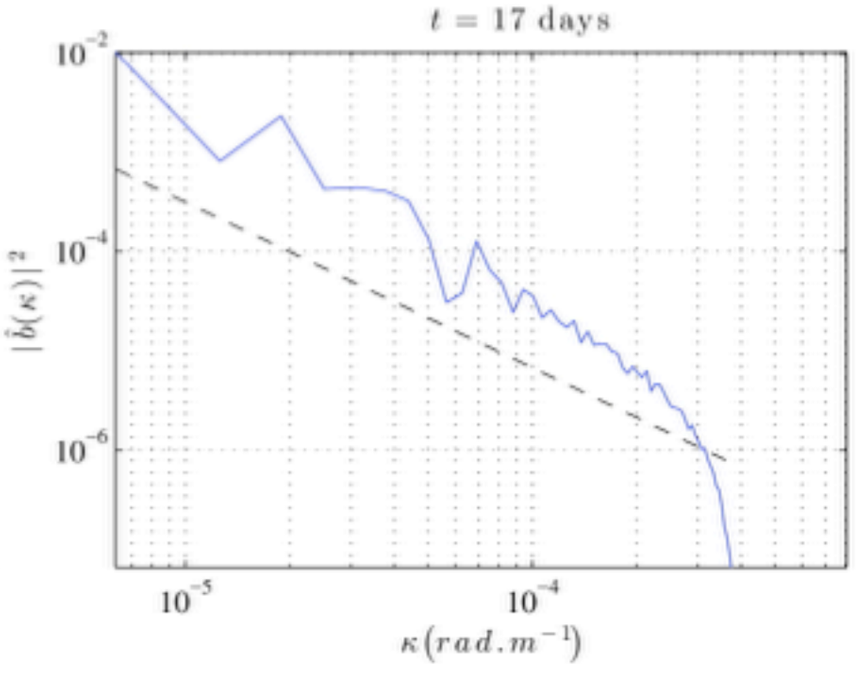




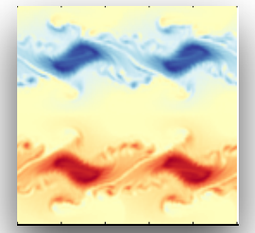
Deterministic 128x128



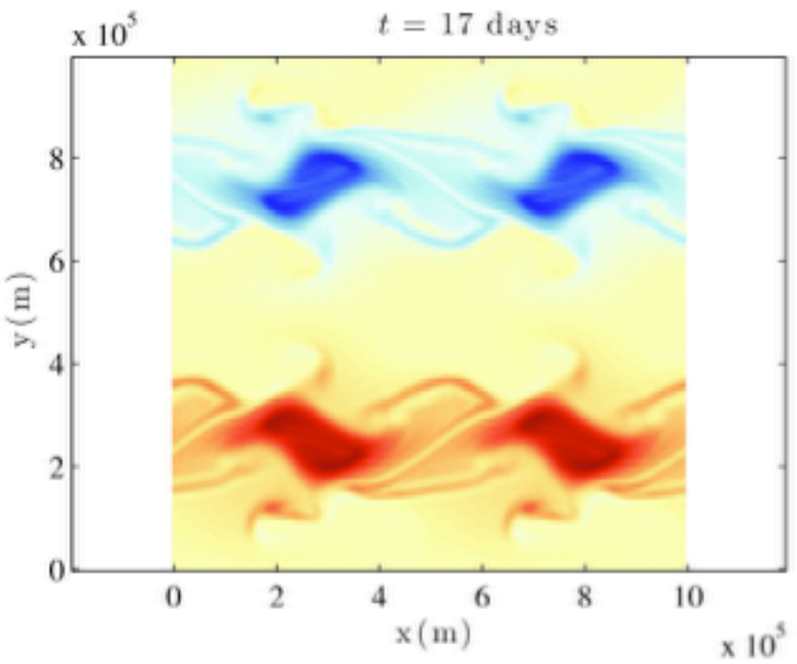
Deterministic 512x512

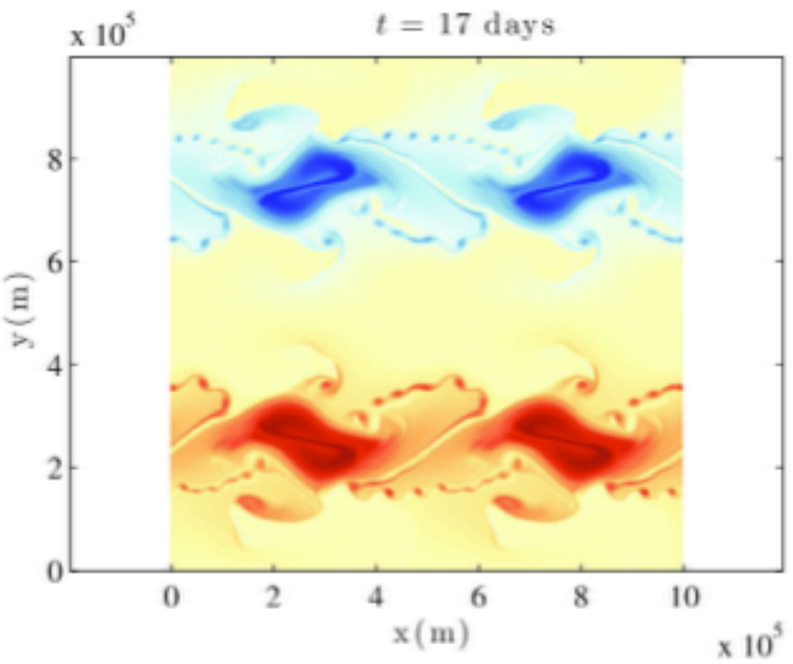


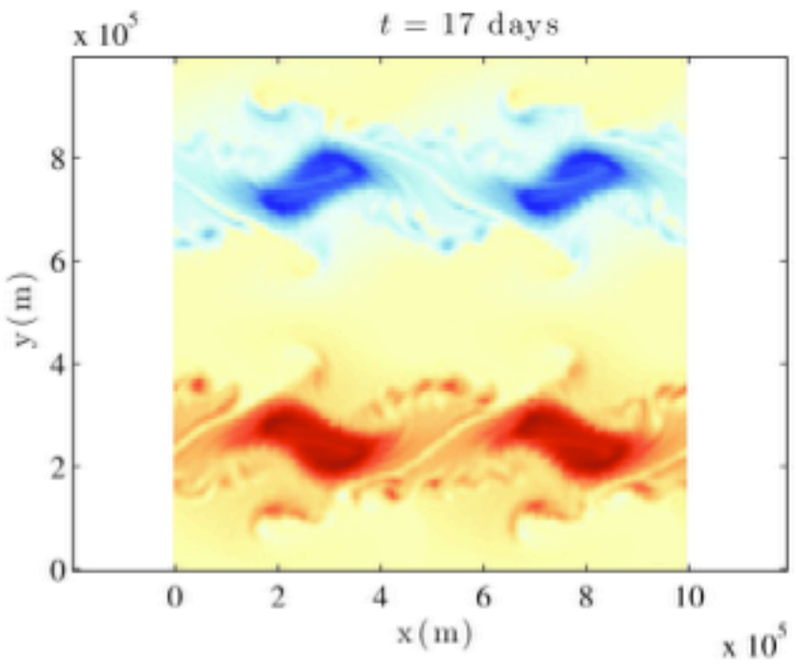
Stochastic 128x128

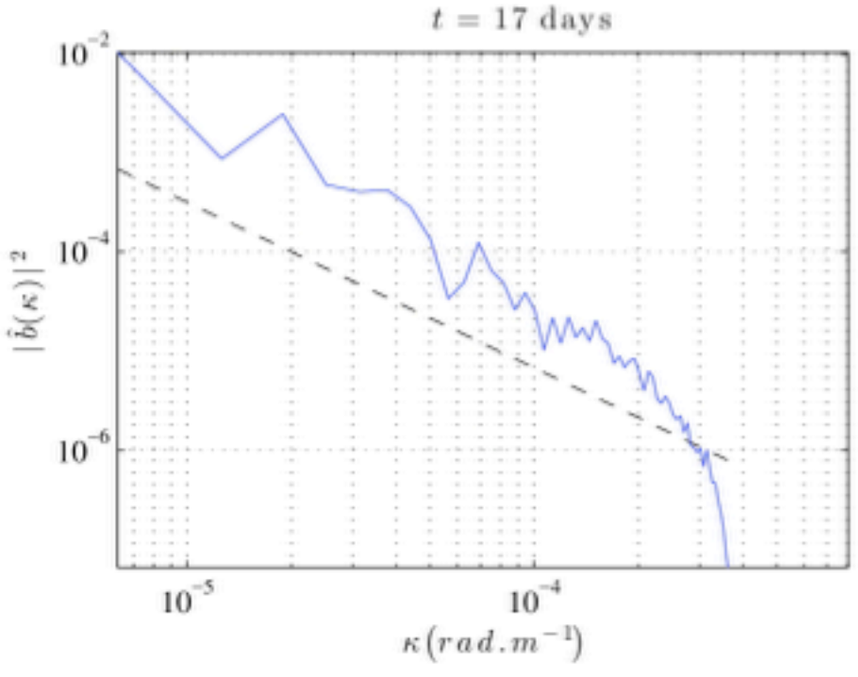


One realization

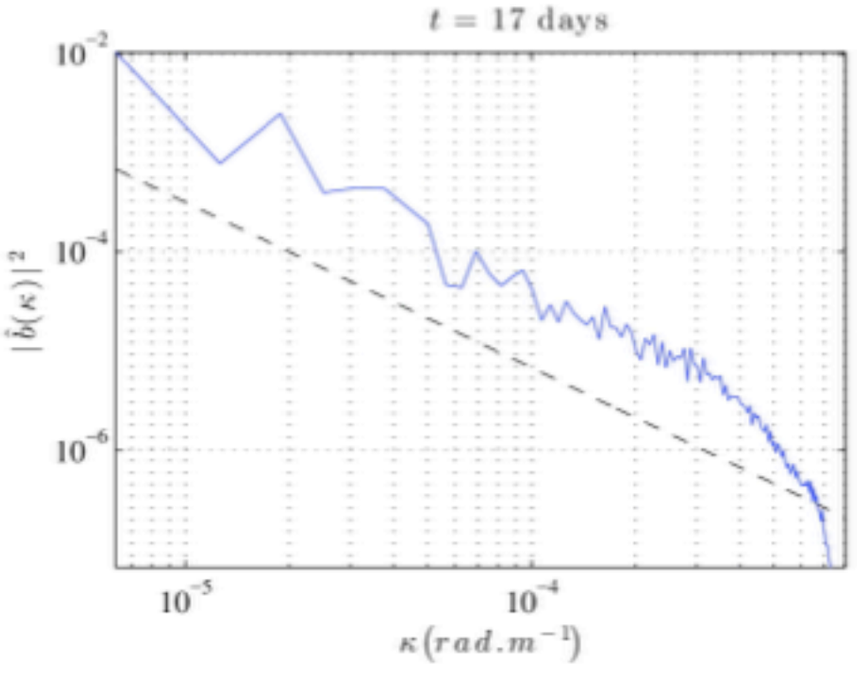




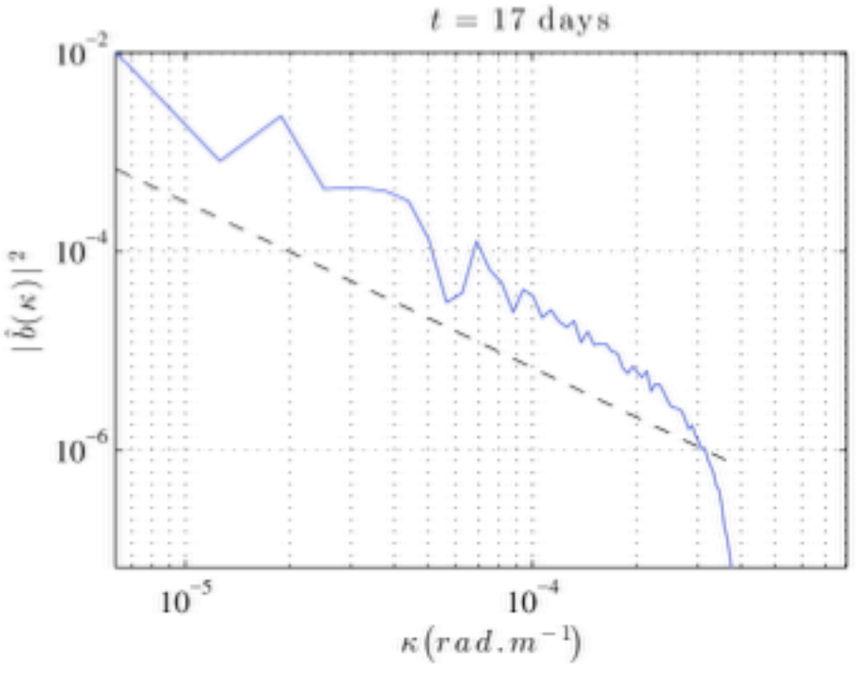




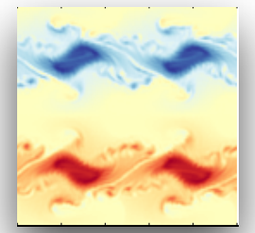
Deterministic 128x128



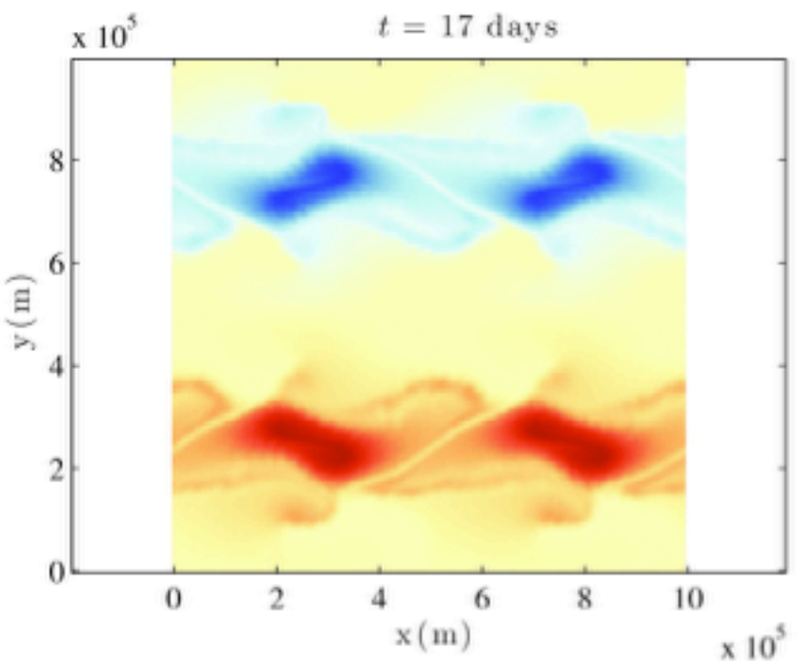
Deterministic 512x512

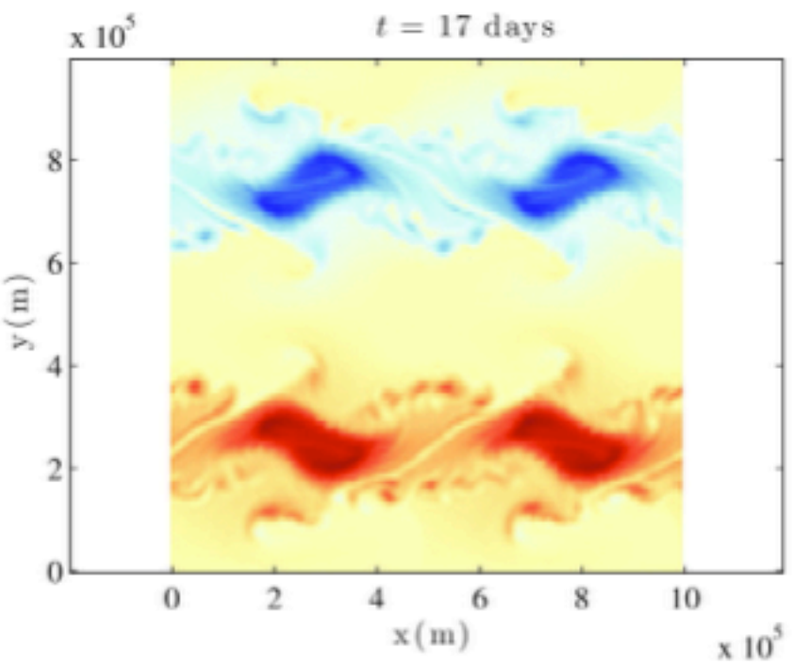


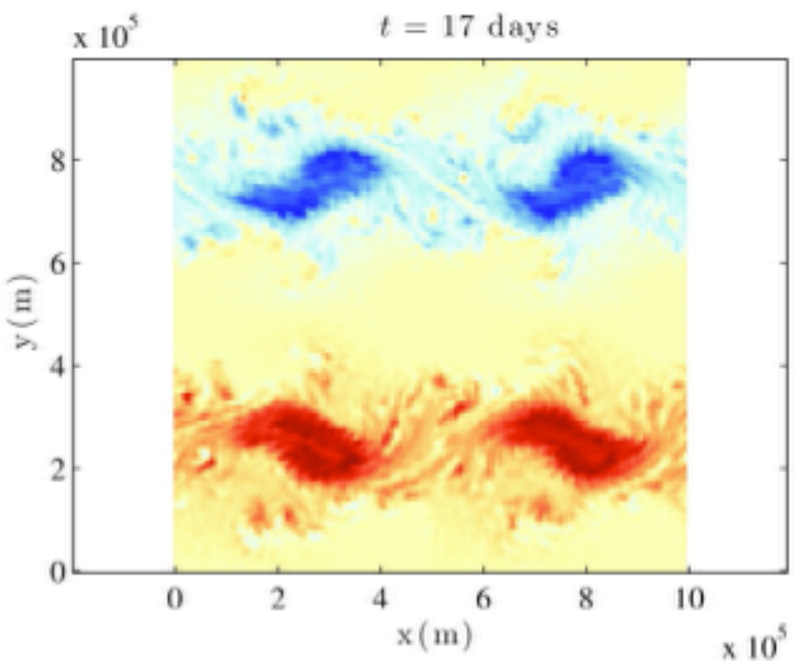
Stochastic 128x128

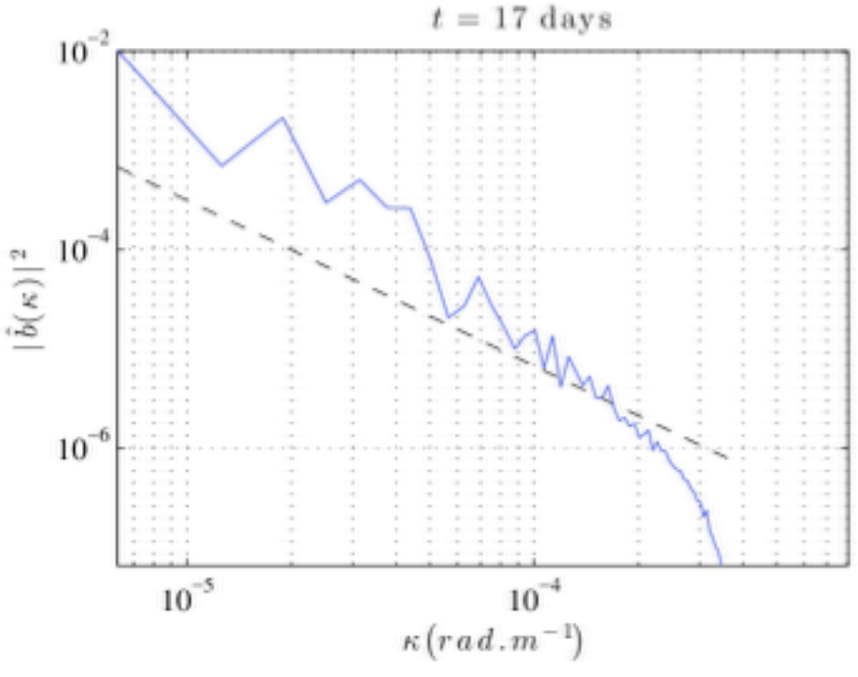


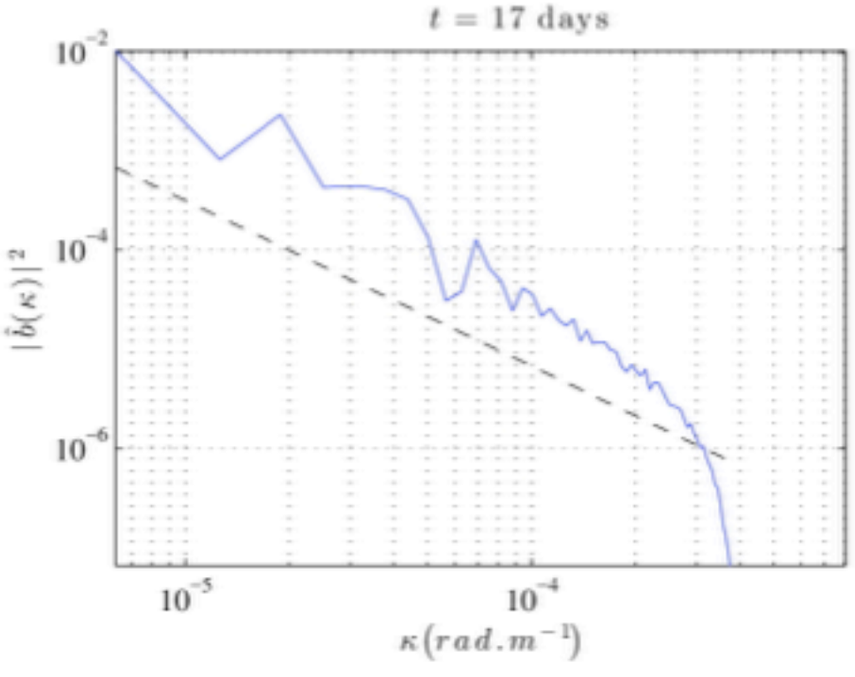
One realization

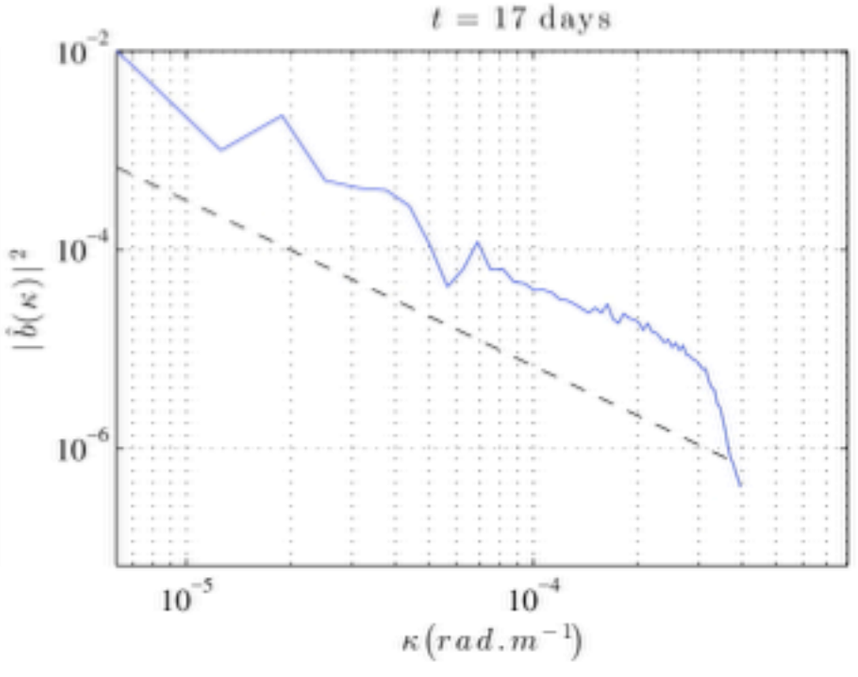




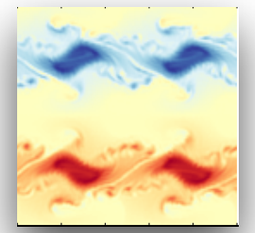




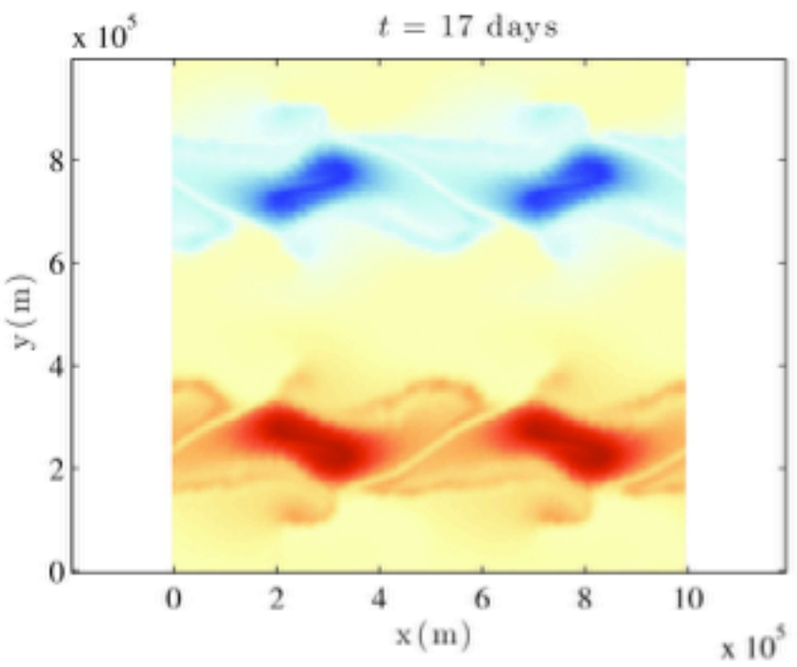


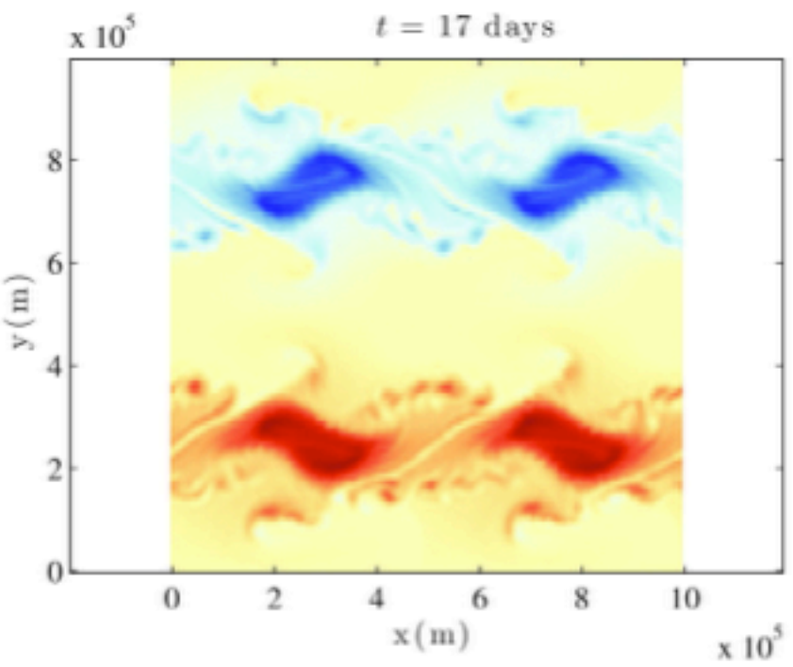


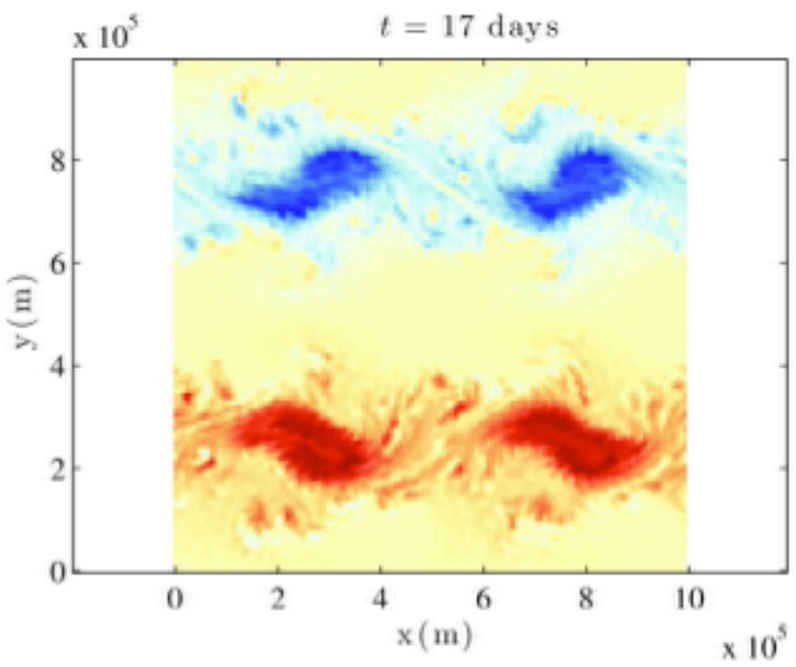
Lower noise **Our model** **Larger noise**

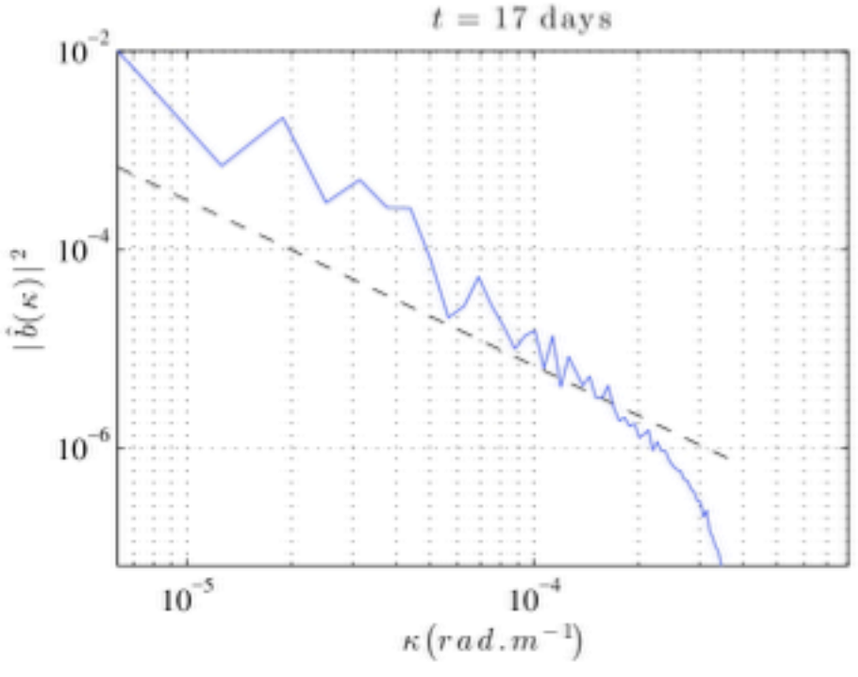


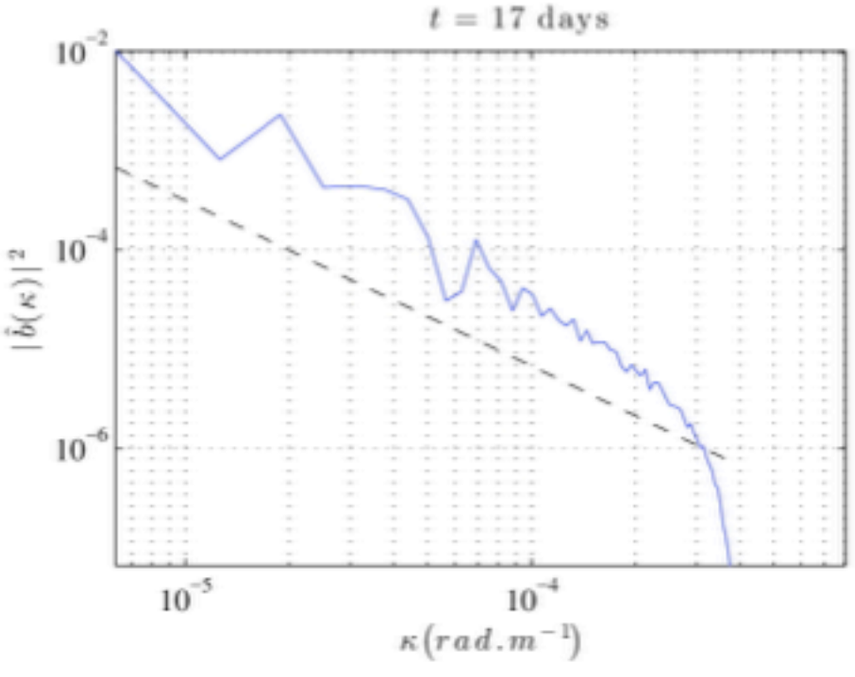
One realization

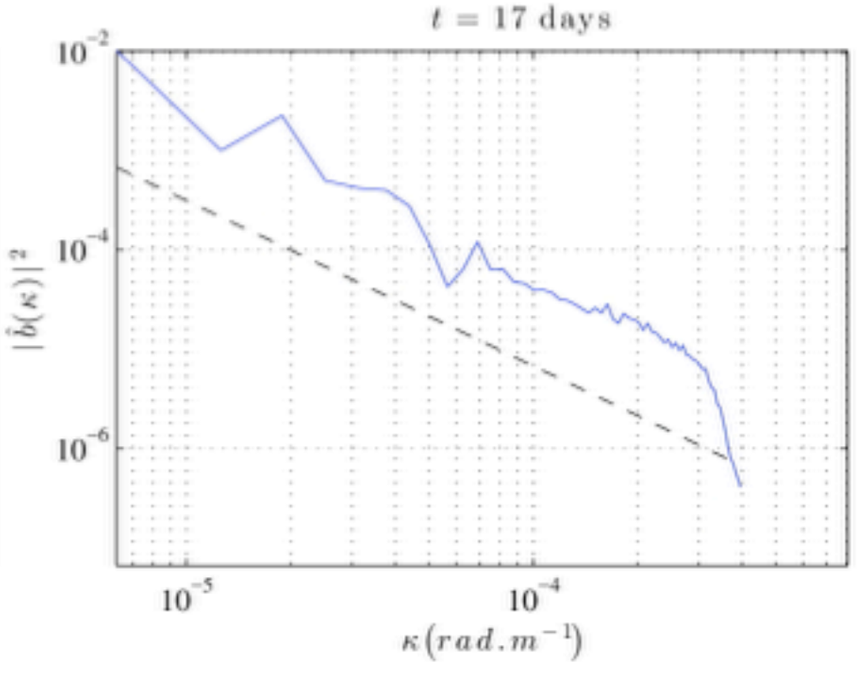




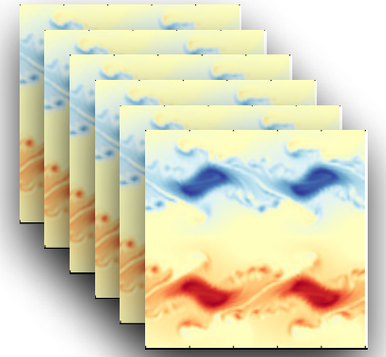




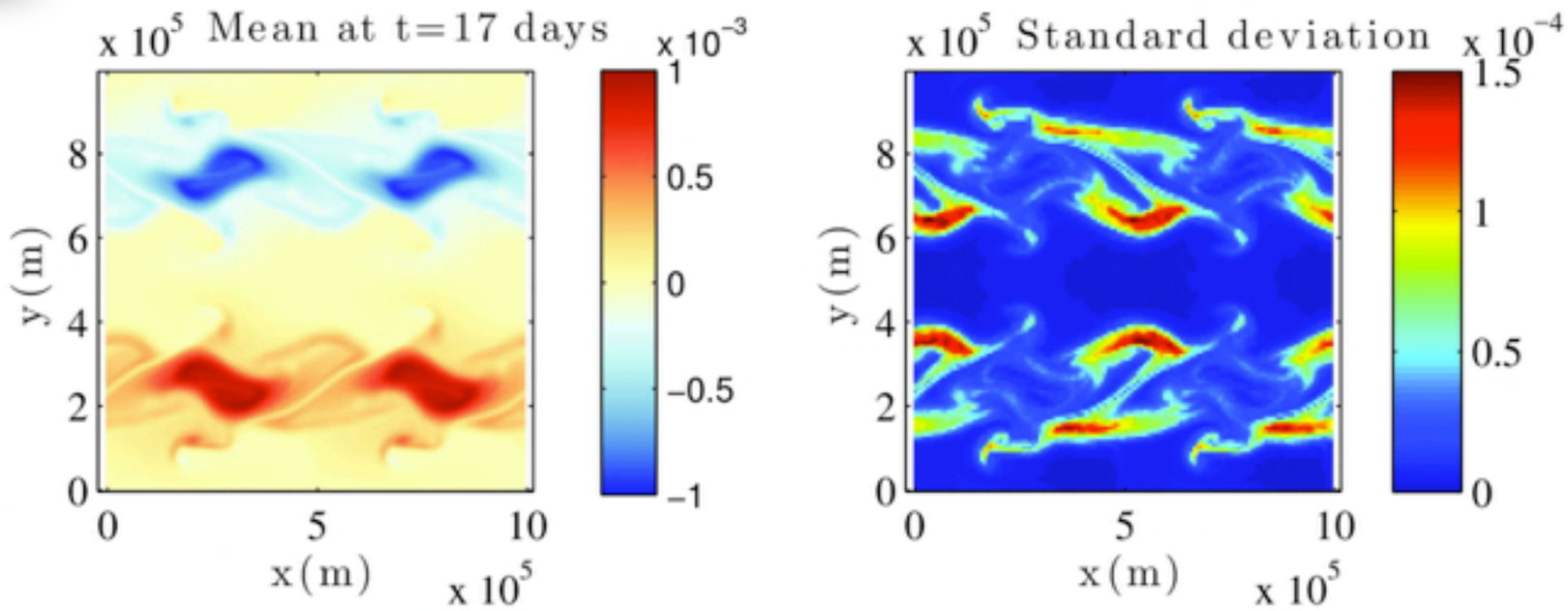


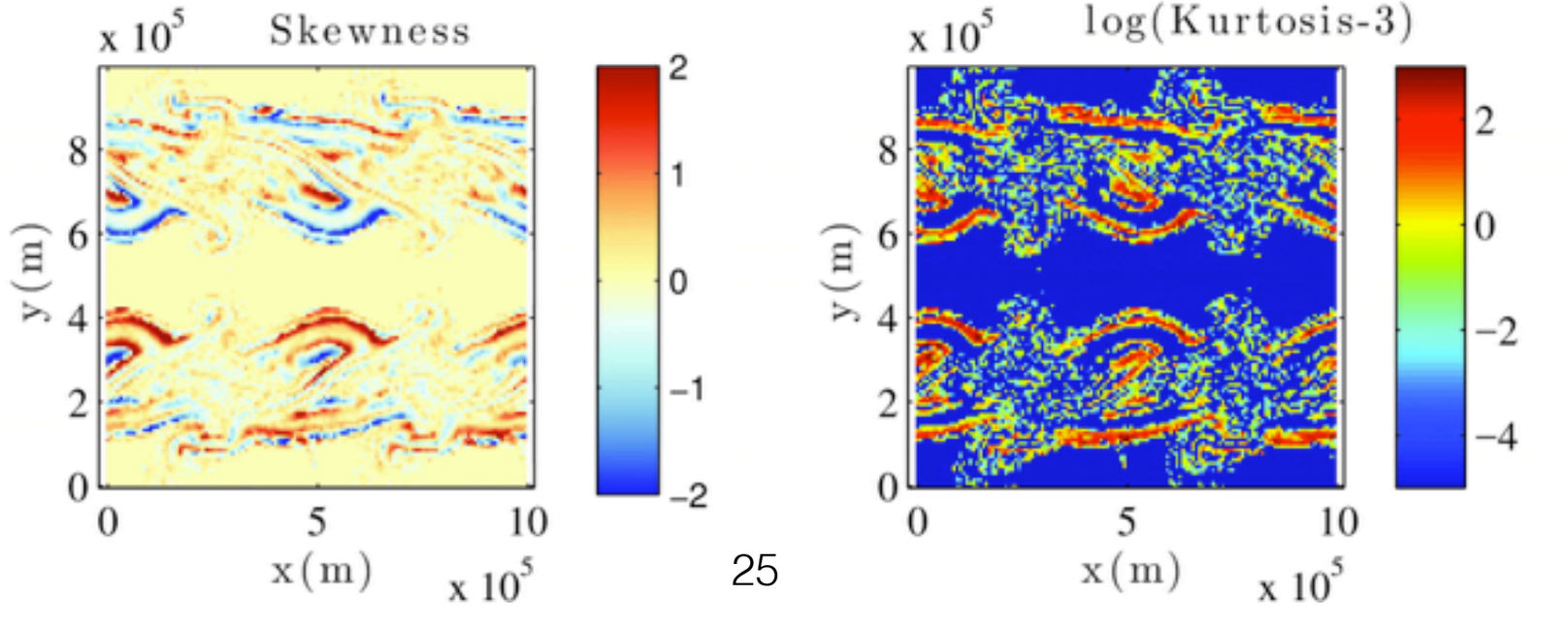


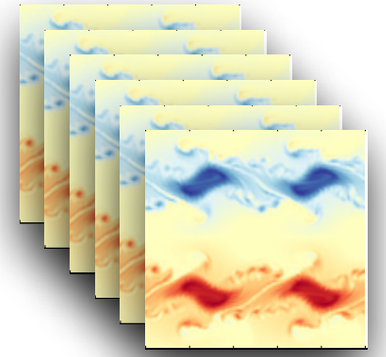
Lower noise **Our model** **Larger noise**



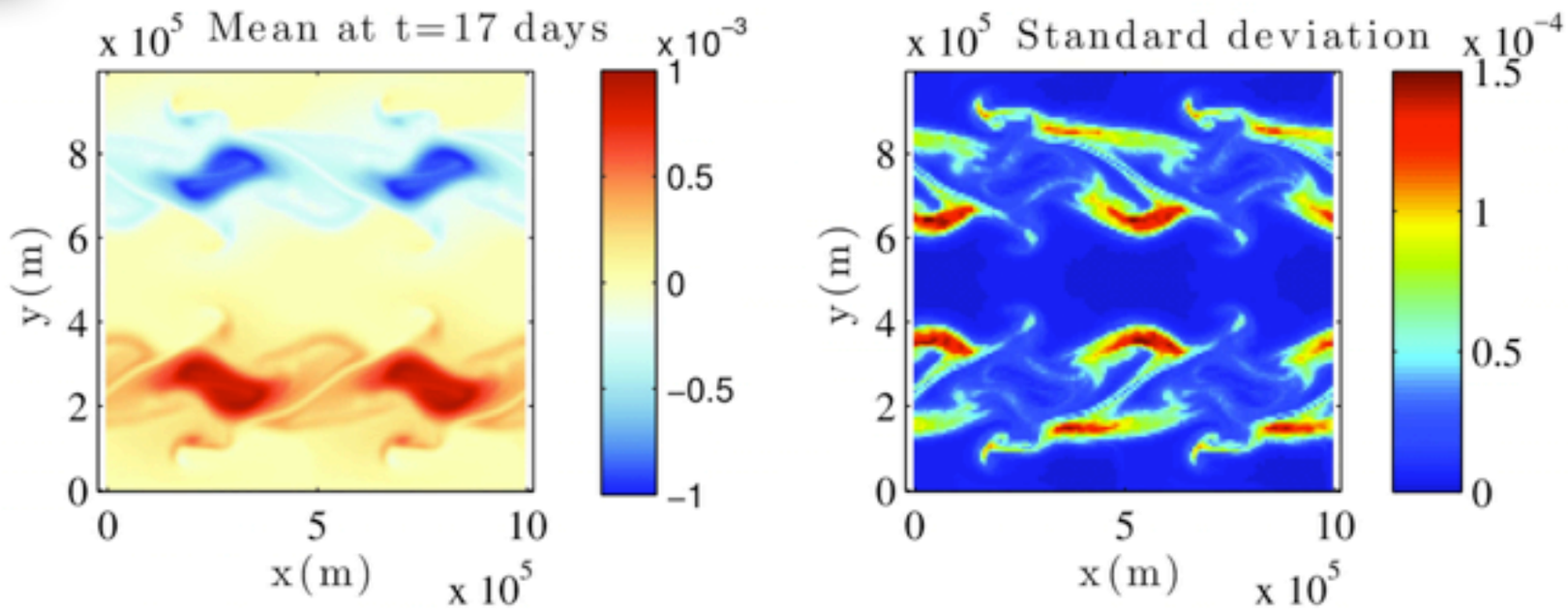
Ensemble

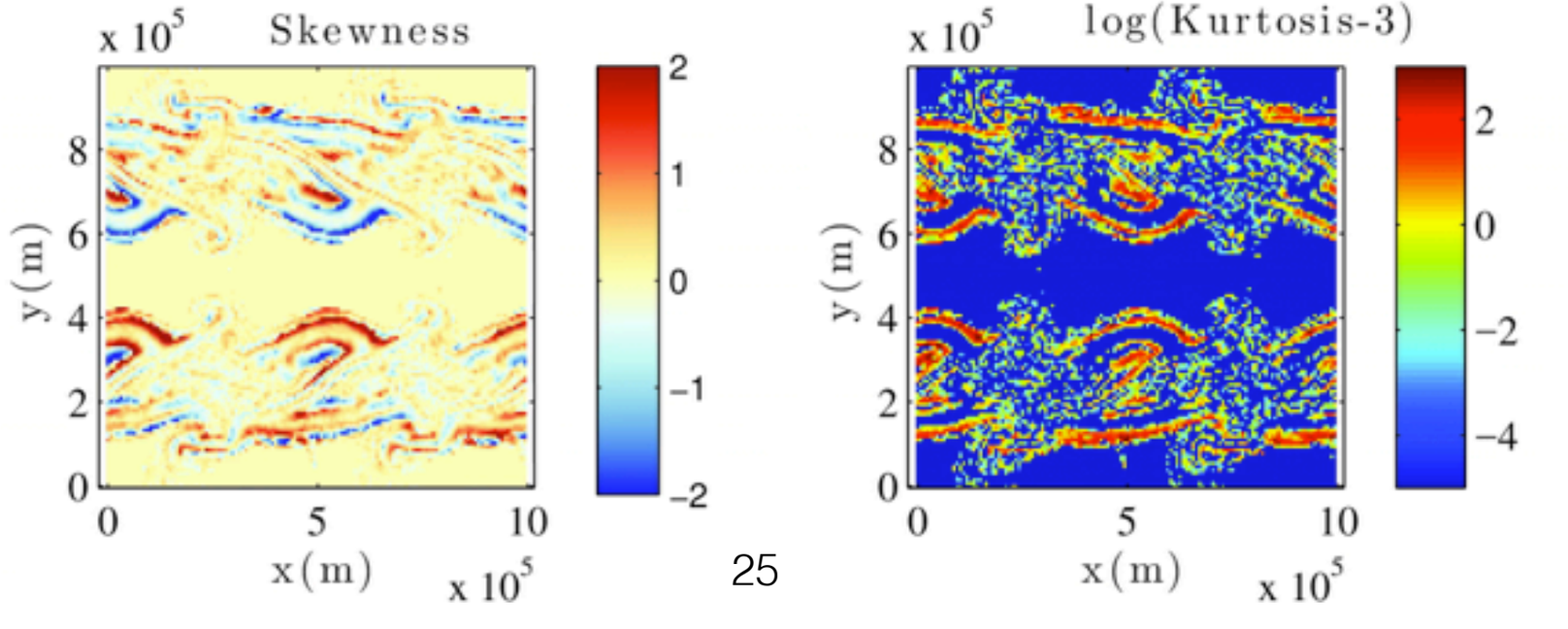

x 10^5 Mean at t=17 days
x 10^-3
y (m)
x (m)
x 10^5
x 10^5 Standard deviation
x 10^-4

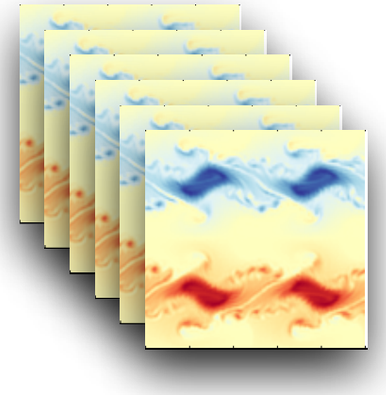

x 10^5 Skewness
log(Kurtosis-3)
y (m)
x (m)
x 10^5



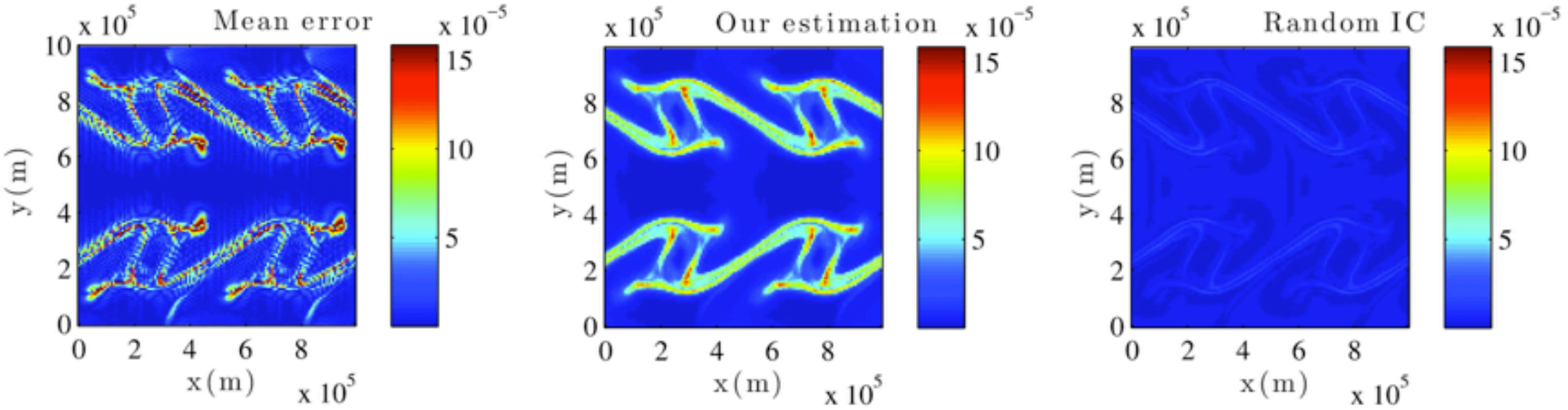
Ensemble

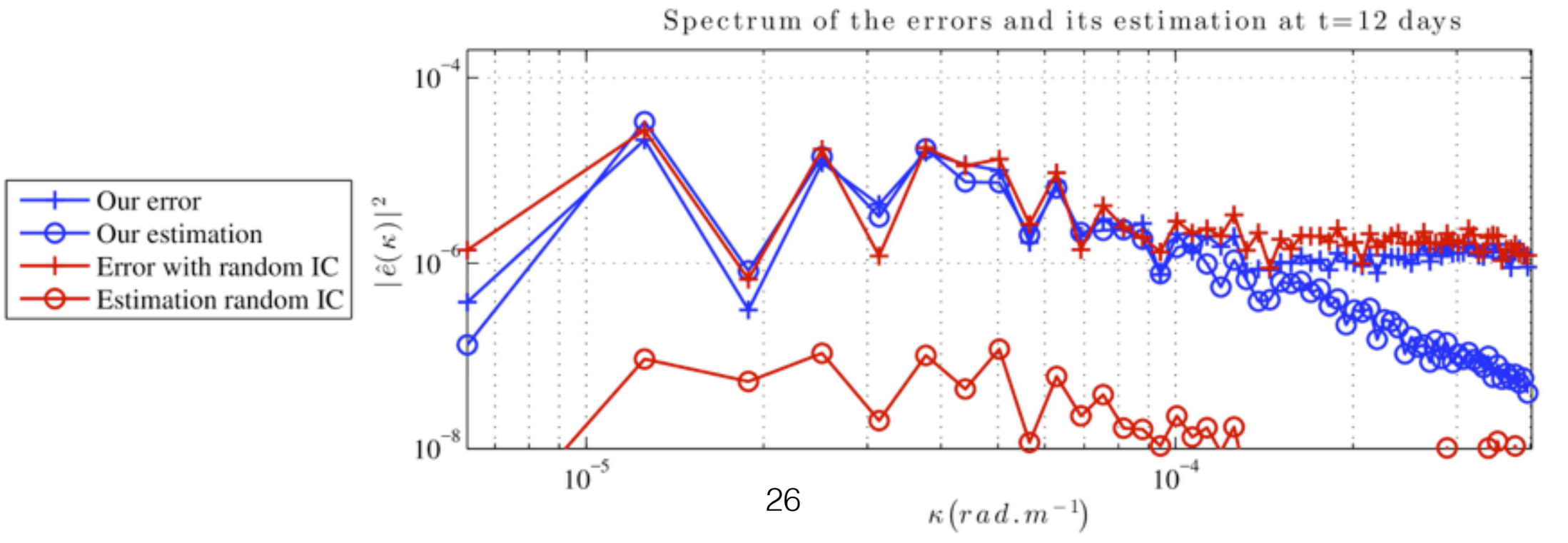

x 10^5 Mean at t=17 days
x 10^-3
y (m)
x (m)
x 10^5
x 10^5 Standard deviation
x 10^-4

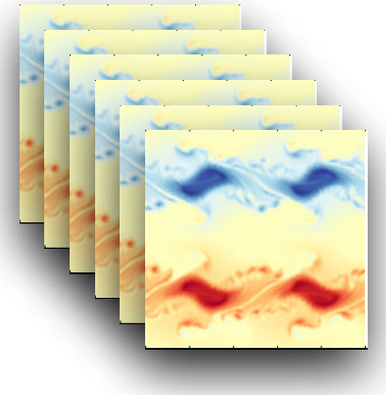

x 10^5 Skewness
log(Kurtosis-3)
y (m)
x (m)
x 10^5



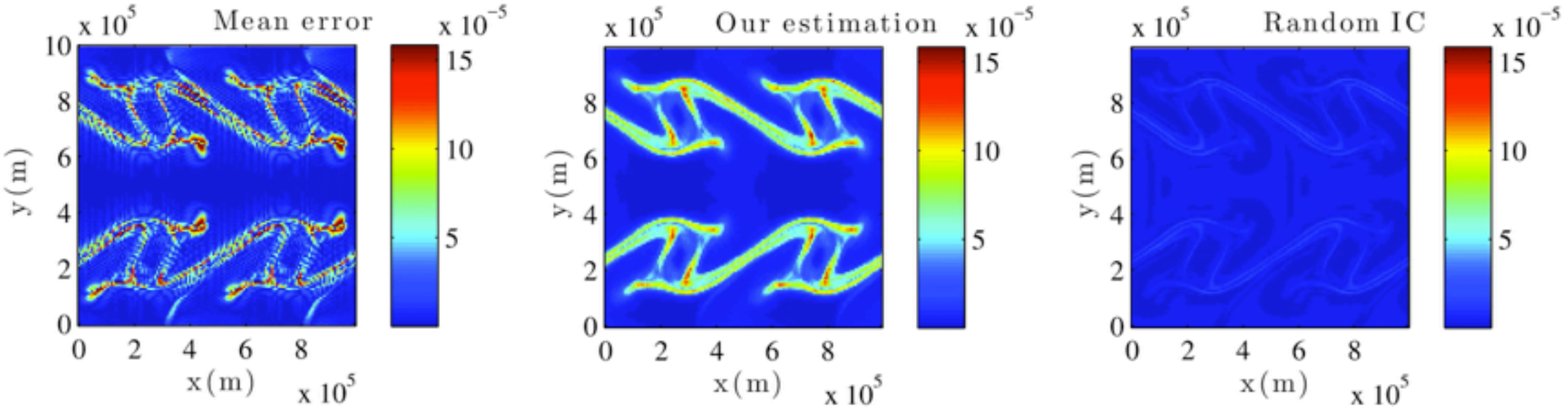
Ensemble

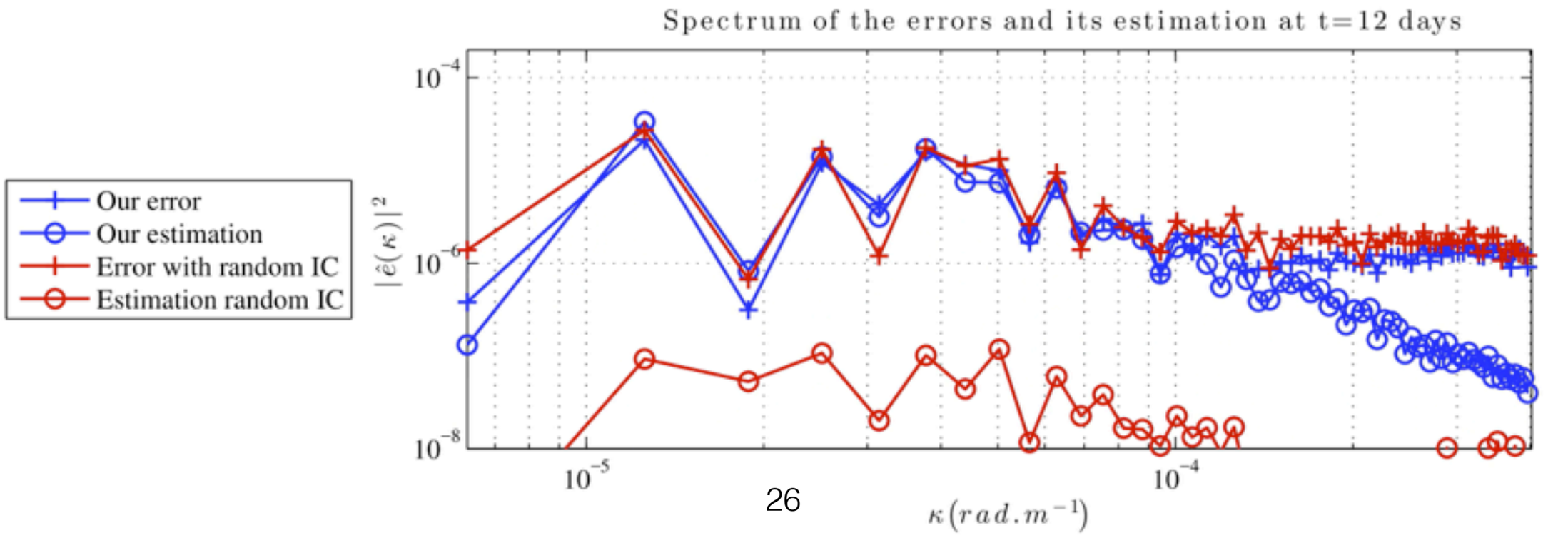

Mean error
Our estimation
Random IC
x 10^5
x 10^-5
y (m)
x (m)
0
2
4
6
8
10
5
10
15


Spectrum of the errors and its estimation at t=12 days
Our error
Our estimation
Error with random IC
Estimation random IC
$|\hat{e}(\kappa)|^2$
$\kappa(rad.m^{-1})$
10^{-4}
10^{-6}
10^{-8}
10^{-5}
10^{-4}



Ensemble


Mean error
Our estimation
Random IC
x 10^5
x 10^-5
y (m)
x (m)
0
2
4
6
8
10
5
10
15


Spectrum of the errors and its estimation at t=12 days
Our error
Our estimation
Error with random IC
Estimation random IC
$|\hat{e}(\kappa)|^2$
$\kappa(rad.m^{-1})$
10^{-4}
10^{-6}
10^{-8}
10^{-5}
10^{-4}

Likely SQG scenarios

tracked by SQG MU

SQG under Strong Uncertainty

SQG SU

Mesoscale divergence

Geostrophic balance

$$\boldsymbol{f} \times \boldsymbol{u} = -\frac{1}{\rho_b} \boldsymbol{\nabla} p'$$

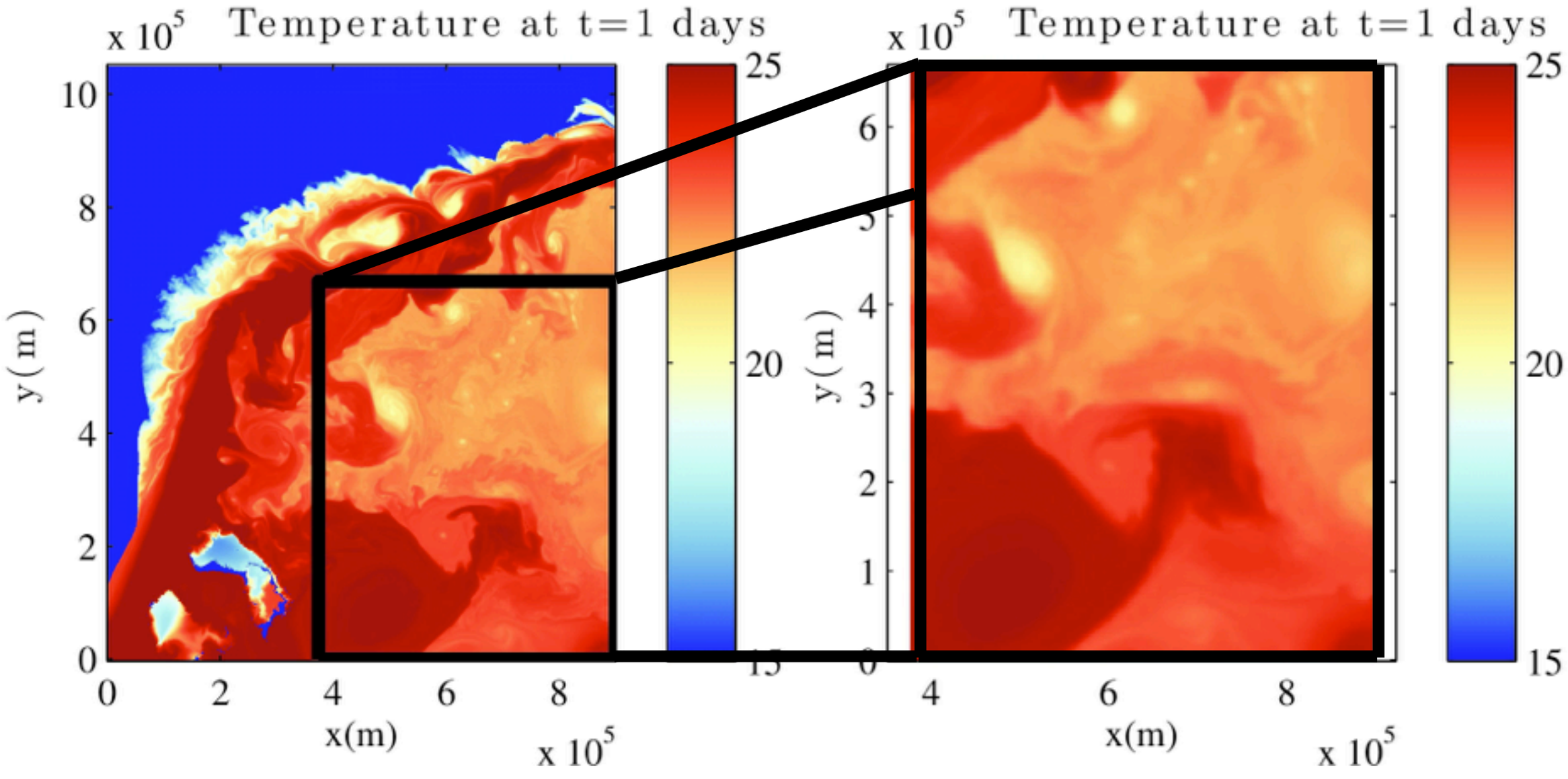
Horizontal Diffusion

$$+ \frac{a}{2} \Delta \boldsymbol{u}$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{u} \propto \Delta \boldsymbol{\nabla}^{\perp} \cdot \boldsymbol{u}$$

Filtering of model outputs:

Gula, Jonathan, M. Jeroen Molemaker, and James C. McWilliams
"Gulf Stream dynamics along the southeastern US seaboard."
Journal of Physical Oceanography 45.3 (2015): 690-715.



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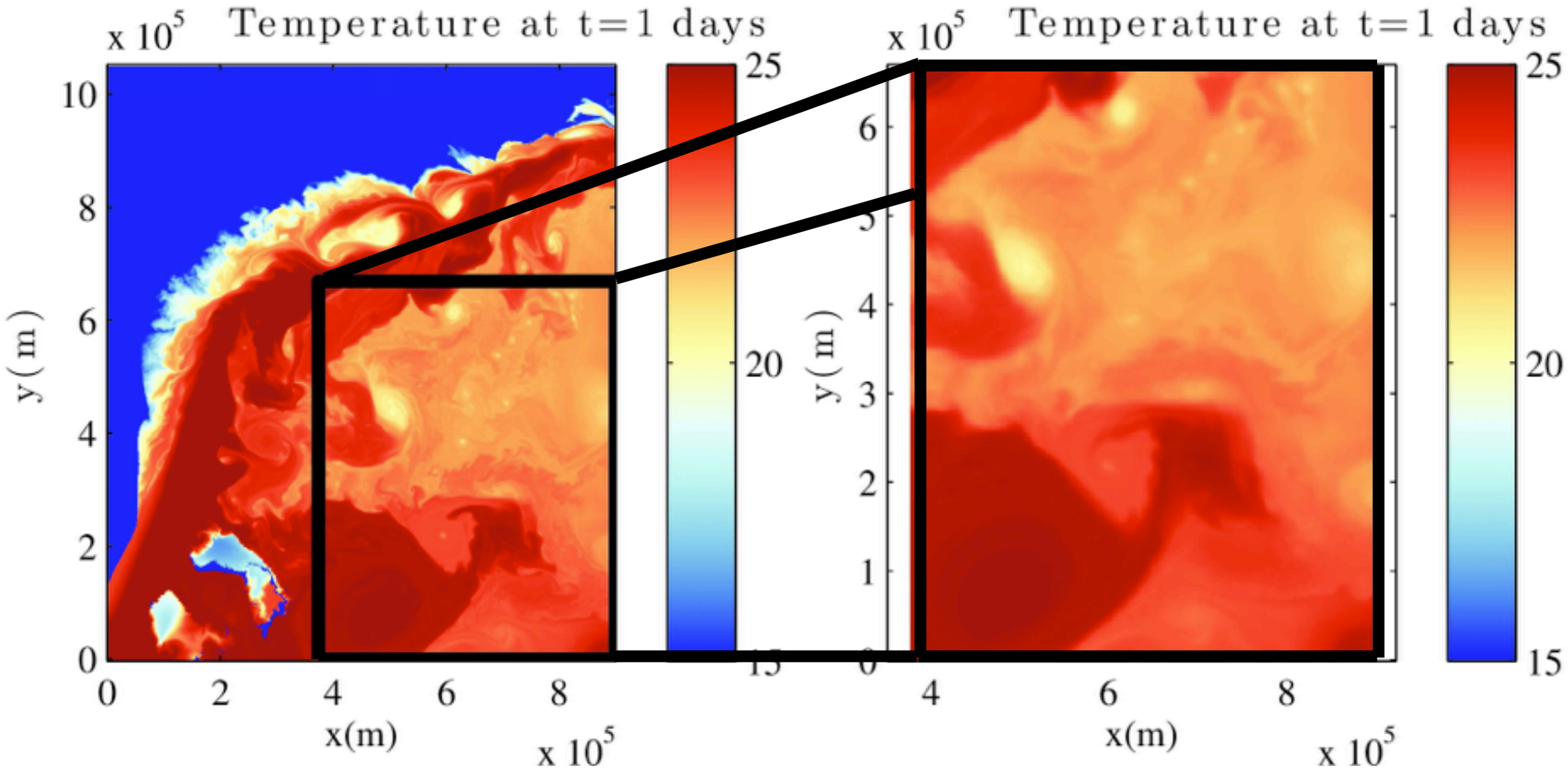
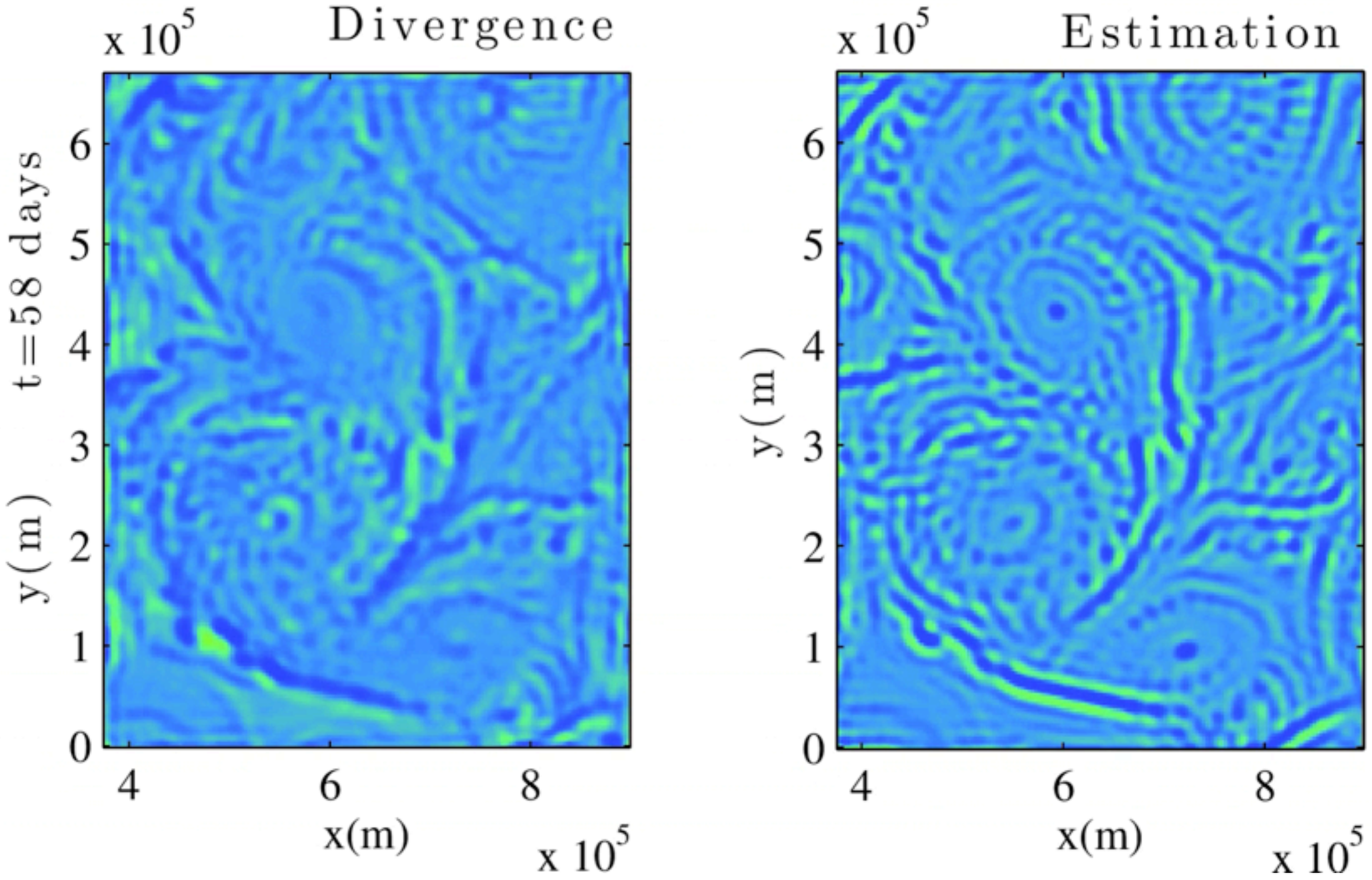
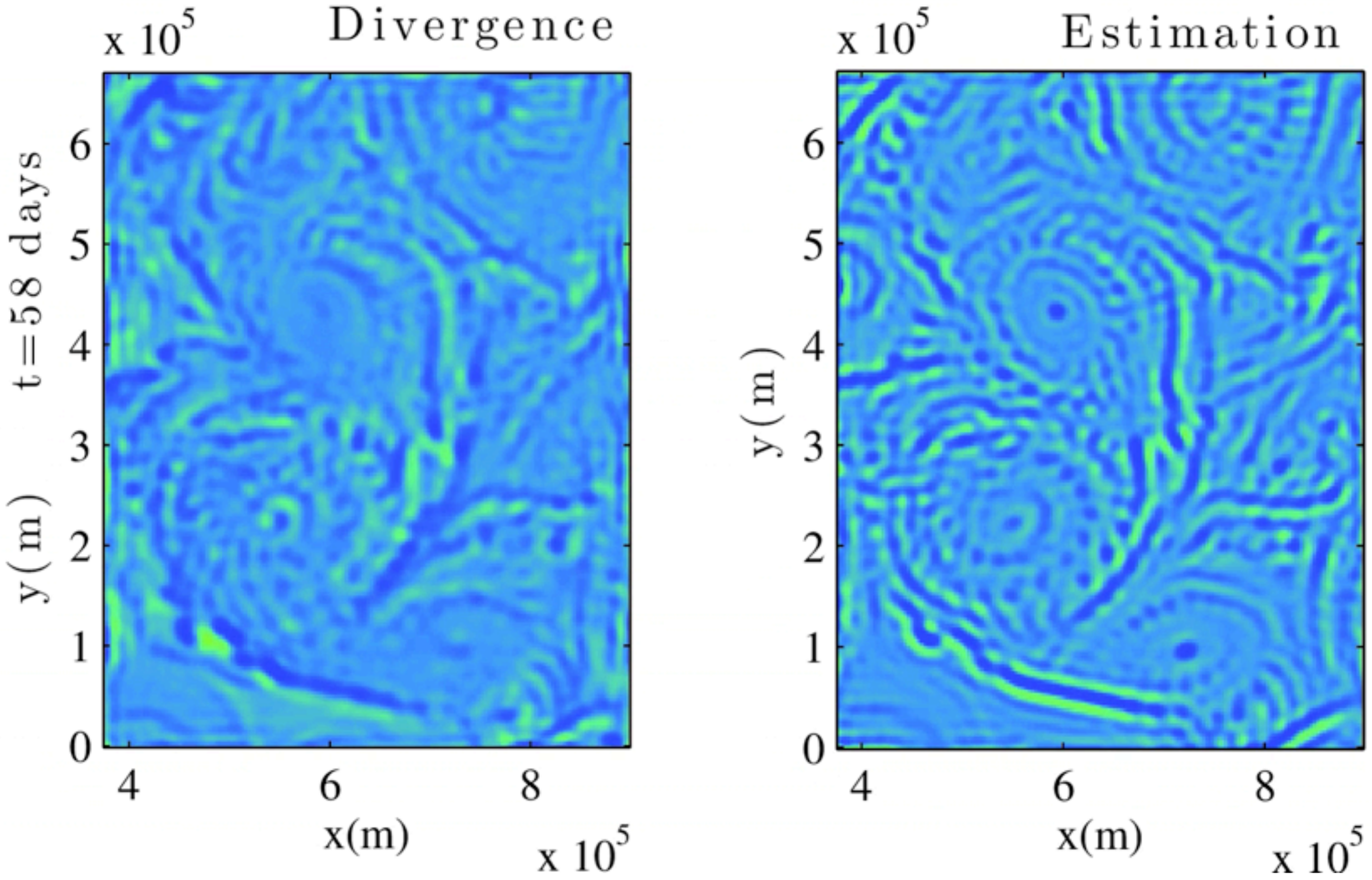


Spatial test



Spatial test



Spectral test